

Edexcel Maths M3

Mark Scheme Pack

2001-2015

# EDEXCEL - LONDON EXAMINATIONS

Stewart House 32 Russell Square London WC1B 5DN

June 2001

Advanced Supplementary/Advanced Level

General Certificate of Education

Subject MECHANICS 6679

Paper No. M3

Question number	Scheme	Marks
1. (a)	$v = \int \frac{1}{2} e^{-\frac{1}{6}t} dt$ $= -3e^{-\frac{1}{6}t} (+c)$ <p>use of limits or <math>t=0, v=10</math></p> $v = 13 - 3e^{-\frac{1}{6}t}$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <math>\rightarrow M1 \leftarrow</math>  A1  M1  A1 </div> (4)
(b)	$t=3, v = 11.2 \text{ ms}^{-1}$	M1 A1 (2)
(c)	13 (ft. if $v = a \pm be^{-\frac{1}{6}t}$ )	B1 + (1) (7)
2. (a)	$\cos \theta = \frac{3}{4}, 0.75, 6/8$	B1 (1)
(b)	$mg \cos \theta (-R) = \frac{mv^2}{0.8}$ $v^2 = 5.88 \text{ m}^2 \text{ s}^{-2}$	M1 A1 A1 (3)
(c)	$\frac{1}{2} m \cdot 5.88 - \frac{1}{2} mu^2 = mg \times 0.2$ $u = 1.4$	M1 A1 A1 (3) (7)
3. (a)	$\frac{1}{2} \times 1.5 v^2 = \frac{52 \times 0.05^2}{2 \times 0.25}$ $v = 0.589 \text{ ms}^{-1} \text{ (3SF)}$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <math>\rightarrow M1 \leftarrow</math> A1 A1  M1 A1 </div> (5)
(b)	$F = 0.6 \times 1.5g$ $\frac{52x}{0.25} \text{ or } \frac{52x}{25}$ $T=F \Rightarrow x = 0.0424 \text{ m or } 4.24 \text{ cm}$ $\text{Min distance} = 0.208 \text{ m or } 20.8 \text{ cm}$	M1 B1 M1 A1 A1 ✓ (5) (10)

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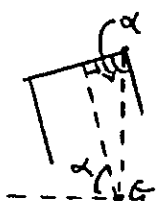
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A.(a)	$g = k/R^2 \Rightarrow k = R^2 g$ $a = -\frac{k}{x^2}$ $v \frac{dv}{dx} = -\frac{R^2 g}{x^2}$	B1 $\rightarrow$ M1 M1 A1 c.s.o. (4)
b)	$\int v dv = -\int \frac{R^2 g}{x^2} dx$ $\frac{v^2}{2} = \frac{R^2 g}{x} (+ c)$ <p style="text-align: center;"><i>correct</i></p> $x=R, v=u \text{ or } \text{use of limits}$ $\frac{v^2}{2} = \frac{R^2 g}{x} + \frac{u^2}{2} - Rg$ <p style="text-align: center;"><i>use of v=0</i></p> $X = \frac{2gR^2}{2gR - u^2}$	$\rightarrow$ M1 A1 $\rightarrow$ M1 A1 M1 A1 (6) (10)
S(a)	$\frac{\pi r^2 h}{6} \quad \frac{1}{6} \pi r^2 h \quad \frac{5}{6} \pi r^2 h$ <p style="text-align: center;">(6) (1) (5)</p> $\frac{1}{2}h \quad \frac{7h}{8} \quad \bar{x}$ $6 \cdot \frac{1}{2}h - \frac{7h}{8} = 5\bar{x}$ $\bar{x} = \frac{17h}{40}$	B2 -1 e.e.o.o. B2 -1 e.e.o.o. M1 A1 A1 (7)
(b)	 $\tan \alpha = \frac{h - \bar{x}}{r}$ <p style="text-align: center;"><i>use of h=4r to obtain expression in h or only</i></p> $\alpha = 66.5^\circ \text{ (1DP)}$	$\rightarrow$ M1 A1 M1 A1 (4) (11)

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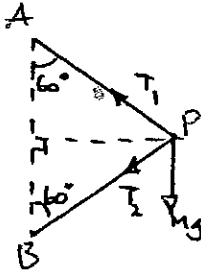
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Question number	Scheme	Marks
6. (a)	$r = \frac{1}{2}h \tan 60^\circ = \frac{\sqrt{3}h}{2} *$	M1 A1 (2)
(b)	 <p> <math>R(1), T_1 \cos 60^\circ - T_2 \cos 60^\circ = mg</math>  <math>R(1), T_1 \sin 60^\circ + T_2 \sin 60^\circ = m \frac{\sqrt{3}}{2} h \omega^2</math>              Use of <math>\cos 60^\circ = \frac{1}{2}</math> and <math>\sin 60^\circ = \frac{\sqrt{3}}{2}</math>              Solving for <math>T_1</math> or <math>T_2</math>  <math>T_1 = \frac{1}{2}m(h\omega^2 + 2g); T_2 = \frac{1}{2}m(h\omega^2 - 2g)</math> </p>	<p> <math>\rightarrow</math> M1 A1  <math>\rightarrow</math> M1 A1                      B1                      M1                      A1; A1 = (8)                 </p>
(c)	$T_2 > 0 \Rightarrow \omega > \sqrt{\frac{2g}{h}}$ $T = \frac{2\pi}{T} \Rightarrow T < 2\pi \sqrt{\frac{h}{2g}} = \frac{\pi\sqrt{2h}}{\sqrt{g}} *$	<p>                     M1 A1 ✓                      M1 A1 c.s.c. (4)                      (14)                 </p>
7. (a)	In equilibrium, $T = mg \sin 30^\circ$ $\lambda \frac{1}{8}a = mg \sin 30^\circ \Rightarrow \lambda = 4mg *$	<p>                     B1                      M1 A1 (3)                 </p>
(b)	$m\ddot{x} = mg \sin 30^\circ - \frac{4mg}{a} \left( \frac{1}{8}a + x \right)$ $\ddot{x} = -\frac{4g}{a}x \Rightarrow \text{SHM}$ $\text{Period} = 2\pi \sqrt{\frac{a}{4g}} = \frac{\pi\sqrt{a}}{\sqrt{g}} *$	<p> <math>\rightarrow</math> M1 A2  <math>\rightarrow</math> M1 A1                      A1 (6)                 </p>
(c)	$\text{Max accel} = \omega^2 a = \frac{4g}{a} \cdot \frac{a}{4} = g$	<p>                     M1 A1 (2)                 </p>
(d)	$x = \frac{a}{4} \sin \omega t; \frac{a}{8} = \frac{a}{4} \sin \omega t$ $\omega t = \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$ $t = \frac{\pi}{2\omega} \sqrt{\frac{a}{g}}$	<p> <math>\rightarrow</math> M1 A1  <math>\rightarrow</math> M1 A1                      A1 ✓ (5)                 </p>
OR:	Circle approach: $\theta = \frac{\pi}{2} - \cos^{-1} \frac{1}{2} = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ $\omega t = \frac{\pi}{6}$ $t = \frac{\pi}{2\omega} \sqrt{\frac{a}{g}}$	<p>                     OR  <math>\rightarrow</math> M1 A1  <math>\rightarrow</math> M1 A1                      A1 ✓ (5)                 </p>
OR:	$\cos^{-1}(-\frac{1}{2}) - \cos^{-1}(0) = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$ $\omega t = \frac{\pi}{6}$ $t = \frac{\pi}{2\omega} \sqrt{\frac{a}{g}}$	<p>                     OR  <math>\rightarrow</math> M1 A1  <math>\rightarrow</math> M1 A1                      A1 ✓ (5)                      (16)                 </p>

Question	Solution	Markscheme
7.(d)	$\theta = \frac{\pi}{2} - \cos^{-1} \frac{1}{2} = \frac{\pi}{6}$ $\omega t = \frac{\pi}{6}$ $t = \frac{\pi}{6} \sqrt{\frac{a}{4g}} = \frac{\pi}{12} \sqrt{\frac{a}{g}}$	$\left[ \begin{array}{l} M1 A1 \\ M1 A1 \end{array} \right]$ <p>A1 f.t.</p>
<u>OR:</u>	$\cos^{-1}(-\frac{1}{2}) - \cos^{-1}(0) = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$ $\omega t = \frac{\pi}{6}$ $t = \frac{\pi}{6} \sqrt{\frac{a}{4g}} = \frac{\pi}{12} \sqrt{\frac{a}{g}}$	$\left[ \begin{array}{l} M1 A1 \\ M1 A1 \end{array} \right]$ <p>A1 f.t.</p>

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\* indicates printed answer

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1.	$0.2a = \frac{5}{x+1}$ $0.2v \frac{dv}{dx} = \frac{5}{x+1}$ $\int v dv = \int \frac{2.5}{x+1} dx$ $\frac{1}{2} v^2 = 2.5 \ln(x+1) (+ C)$ $x=0, v=5 \Rightarrow C = 12.5$ $\frac{2.5}{2} = 2.5 \ln(x+1) + 12.5$ $x = 53.6 \text{ (3sf)}$	<p>M1</p> <p>→ M1</p> <p>→ M1</p> <p>A1 A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>(8)</p>
2. (a)	$PE \text{ Loss} = 0.5g(2+x); EPE_{at C} = \frac{19.6x^2}{4}$ $0.5g(2+x) = \frac{19.6x^2}{4}$ $k(x^2 - 2x - 2) = 0$ <p>solving</p> $AC = 4m$	<p>B1; B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 ✓</p> <p>(6)</p>
(b)	$T_c = \frac{19.6 \times 2}{2} = 19.6$ $19.6 - 0.5g = 0.5a$ $a = 29.4 \text{ ms}^{-2}$	<p>B1 ✓</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>(9)</p>
3. (a)	Line of action of weight must pass through C which is not above centre of rod (or equivalent)	<p>B1</p> <p>(1)</p>
(b)	<p>Method A:</p> <p>R(along AC): <math>T_1 = 2mg \sin \alpha = \frac{6mg}{5}</math></p> <p>R(along BC): <math>T_2 = 2mg \cos \alpha = \frac{8mg}{5}</math> *</p> <p>[Equiv. to moments about A, B respectively]</p> <p>OR Method B: R(A), <math>T_1 \sin \alpha + T_2 \cos \alpha = 2mg</math></p> <p>t(→), <math>T_1 \cos \alpha = T_2 \sin \alpha</math></p> <p>solving to find <math>T_1</math> or <math>T_2</math></p> <p><math>T_1 = \frac{6mg}{5}; T_2 = \frac{8mg}{5}</math> *</p>	<p>M1 M1 A1</p> <p>M1 A1</p> <p>→ M1</p> <p>→ M1</p> <p>M1</p> <p>A1; A1</p> <p>(5)</p>
(c)	$\frac{8mg}{5} = \frac{mg(BC - a)}{a}$ $BC = 2a \sin \alpha$ $k = 8$	<p>M1 A1</p> <p>B1</p> <p>A1</p> <p>(4)</p> <p>(10)</p>

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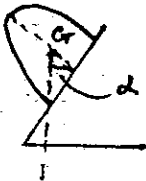
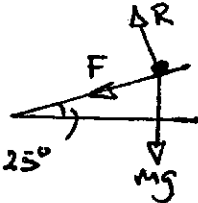
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4.(a)	$\int_0^r (\pi) y^2 x dx = \pi \int_0^r (\pi) y^2 dx$ $\int_0^r r x^2 dx = \pi \int_0^r r x dx$ $\left[ (r) \frac{x^3}{3} \right]_0^r = \pi \left[ (r) \frac{x^2}{2} \right]_0^r$ $\bar{x} = 2r/3$	$\rightarrow$ M1 A1 $\left[ \begin{array}{l} \pi \\ \pi \end{array} \right]$ A1 A1 A1 (5)
(b)	 <p>vertical thro' CM and lowest point of plane face</p> <p><math>\tan \alpha = r/r/3</math></p> <p><math>\alpha = 72^\circ</math> (nearest degree)</p>	M1 M1 A1 A1 (4)
5.	 <p>R(1), <math>R \sin 25^\circ - F \sin 25^\circ = mg</math></p> <p>R(1), <math>R \sin 25^\circ + F \sin 25^\circ = \frac{mv^2}{40}</math></p> <p><math>F = 0.6R</math> used</p> <p>Eliminating R</p> <p>Solving for v</p> <p><math>v = 24.1 \text{ ms}^{-1}, 24 \text{ ms}^{-1}</math></p>	$\rightarrow$ M1 A2 $\rightarrow$ M1 A2 M1 $\rightarrow$ M1 $\left[ \begin{array}{l} M1 \\ M1 \end{array} \right]$ A1 (10)
6.(a)	<p>If SHM, <math>a = 1.2</math></p> <p>Using <math>v^2 = \omega^2(a^2 - x^2)</math></p> <p><math>0.27 = \omega^2(1.2^2 - 0.6^2)</math> or <math>0.2 = \omega^2(1.2^2 - 0.8^2)</math></p> <p>Solve for <math>\omega (= 0.5)</math> and use in other eqn</p> <p>Shown to be correct</p>	B1 $\rightarrow$ M1 $\left[ \begin{array}{l} A1 \\ M1 \end{array} \right]$ A1 c.s.o. (5)
(b)	$v = a\omega = 1.2 \times 0.5 = 0.6$	M1 A1 (2)
(c)	$ x  = \omega^2 \times 0.6 = 0.15 \text{ ms}^{-2}$	M1 A1 ✓ (2)
(d)	<p><math>0.6 = a \sin \omega t</math> or <math>0.8 = a \sin \omega t</math></p> <p><math>t = \frac{1}{\omega} \left( \sin^{-1} \frac{0.6}{a} - \sin^{-1} \frac{0.6}{a} \right)</math></p> <p><math>= 0.412 \text{ s}</math> (3sf)</p>	M1 M1 A1 ✓ A1 (4)

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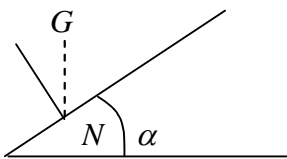
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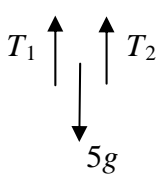
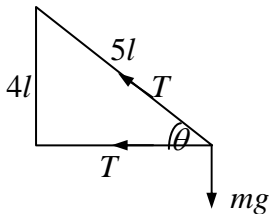
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7.(a)	$\frac{1}{2} m \frac{7as}{2} - \frac{1}{2} m v^2 = mga$ $(\leftarrow), R = \frac{mv^2}{a} = \frac{3mg}{-2}$	<p>M1 A1 M1 A1 (4)</p>
(b)	$\frac{1}{2} m \frac{7as}{2} - \frac{1}{2} m v^2 = mga (1 + \cos \theta)$ $(\leftarrow), mg \cos \theta = \frac{mv^2}{a}$ <p>Eliminating <math>v^2</math></p> <p>Solving to give <math>\cos \theta = \frac{1}{2}, \theta = 60^\circ *</math></p>	<p>M1 A1 M1 A1 M1 A1 (7)</p>
(c)	$v \cos 60^\circ t = a \sin 60^\circ$ $v^2 = a g \cos 60^\circ$ <p>Making <math>t</math> explicit</p> $t = \sqrt{\frac{6a}{g}}$	<p>M1 B1 M1 A1 (4)</p>
		(15)



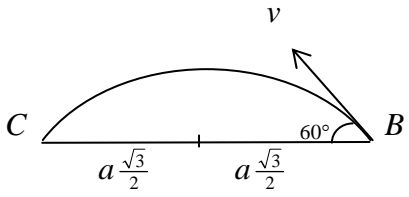
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1.	<p>(a) <math>a = 0.25</math></p> <p><math>\frac{2\pi}{\omega} = 2 \Rightarrow \omega = \pi</math></p> <p><math>-0.125 = 0.25 \cos \omega t</math></p> <p>(b) <math>t = \frac{1}{\pi} \cos^{-1}(-0.5)</math></p> <p><math>= \frac{2}{3}</math></p>	<p>B1</p> <p>B1</p> <p>M1A1</p> <p>M1</p> <p>A1 (6)</p> <p>(6 marks)</p>
2.	<p>(a) <math>(\uparrow) 3mg \cos \alpha^\circ = mg</math></p> <p><math>\alpha = \cos^{-1}(\frac{1}{3})</math></p> <p><math>= 70.5</math></p> <p>(b) <math>(\leftarrow) 3mg \sin \alpha = mr \times 2gk</math></p> <p><math>l \sin \alpha = r</math></p> <p><math>l = \frac{3}{2}k</math></p>	<p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>B1</p> <p>M1 A1 (5)</p> <p>(9 marks)</p>
3.	<p>(a) <math>2e^{-0.1x} = 2.5a</math></p> <p><math>\frac{4}{5}e^{-0.1x} = v \frac{dv}{dx}</math></p> <p><math>-8e^{-0.1x} = \frac{1}{2}v^2 (+c)</math></p> <p><math>x = 0, v = 2 \Rightarrow c = -10</math></p> <p><math>v^2 = 20 - 16e^{-0.1x}</math></p> <p>(b) <math>16 = 20 - 16e^{-0.1x} \Rightarrow e^{-0.1x} = \frac{1}{4}</math></p> <p><math>0.1x = \ln 4</math></p> <p><math>x = 13.9</math></p> <p>(c) Appropriate comment.</p>	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (6)</p> <p>M1</p> <p>M1</p> <p>A1 (3)</p> <p>B1 (1)</p> <p>(10 marks)</p>

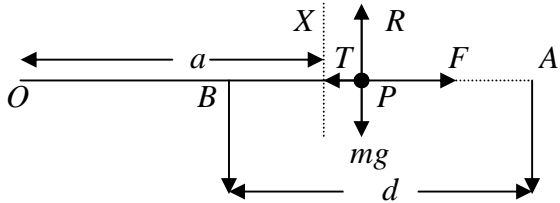
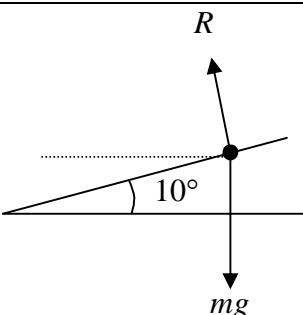
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4.	<p>(a) <math>\frac{1}{2} \times 0.2 \times 5^2 - \frac{1}{2} \times 0.2 \times u^2 = \frac{1}{2} \times \frac{20(0.5)^2}{1.5}</math></p> <p><math>u^2 = \frac{25}{3}</math></p> <p><math>u = 2.89 \text{ ms}^{-1}</math></p> <p>(b) <math>\frac{1}{2} \times 0.2 \times 5^2 - \frac{1}{2} \times 0.2 \times 1.5^2 = \frac{1}{2} \times \frac{20x^2}{1.5}</math></p> <p><math>x^2 = 0.34125</math></p> <p><math>T = \frac{20x}{1.5} = 7.8 \text{ N}</math></p>	<p>M1 A1 A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 (5)</p> <p>(10 marks)</p>															
5.	<p>(a)</p> <table border="0"> <tr> <td>Cone</td> <td>Cylinder</td> <td>Whole</td> </tr> <tr> <td><math>\frac{1}{3} \pi (2r)^2 h</math></td> <td><math>\pi r^2 h</math></td> <td><math>\frac{1}{3} \pi (2r)^2 h + \pi r^2 h</math></td> </tr> <tr> <td>(4)</td> <td>(3)</td> <td>(7)</td> </tr> <tr> <td><math>\frac{1}{4} h</math></td> <td><math>\frac{1}{2} h</math></td> <td><math>\bar{x}</math></td> </tr> <tr> <td><math>-4 \times \frac{1}{4} h</math></td> <td><math>+ 3 \times \frac{1}{2} h</math></td> <td><math>= 7 \bar{x}</math></td> </tr> </table> <p><math>\bar{x} = \frac{1}{14} h</math></p> <p>(b)</p>  <p>Use of <math>G</math> above <math>N</math></p> <p><math>\tan \alpha = \frac{r}{h - \frac{1}{14} h} = \frac{7}{26}</math></p> <p><math>r = \frac{1}{4} h</math></p>	Cone	Cylinder	Whole	$\frac{1}{3} \pi (2r)^2 h$	$\pi r^2 h$	$\frac{1}{3} \pi (2r)^2 h + \pi r^2 h$	(4)	(3)	(7)	$\frac{1}{4} h$	$\frac{1}{2} h$	$\bar{x}$	$-4 \times \frac{1}{4} h$	$+ 3 \times \frac{1}{2} h$	$= 7 \bar{x}$	<p>M1 A1</p> <p>B1 B1</p> <p>M1 A1</p> <p>M1 A1 cso (8)</p> <p>M1</p> <p>M1 A1</p> <p>A1 (4)</p> <p>(12 marks)</p>
Cone	Cylinder	Whole															
$\frac{1}{3} \pi (2r)^2 h$	$\pi r^2 h$	$\frac{1}{3} \pi (2r)^2 h + \pi r^2 h$															
(4)	(3)	(7)															
$\frac{1}{4} h$	$\frac{1}{2} h$	$\bar{x}$															
$-4 \times \frac{1}{4} h$	$+ 3 \times \frac{1}{2} h$	$= 7 \bar{x}$															

Question Number	Scheme	Marks
6.	<p>(a) <math>mg = \frac{8mge}{4a}</math></p> <p><math>\frac{9}{2}a = AO</math></p> <p>(b) <math>mg - \frac{8mg}{4a}(e+x) = m\ddot{x}</math></p> <p><math>\ddot{x} = -\frac{2g}{a}x</math></p> <p><math>T = 2\pi\sqrt{\frac{a}{2g}} = \pi\sqrt{\frac{2a}{g}} \quad (*)</math></p> <p><math>v = d\omega</math></p> <p><math>\frac{1}{2}\sqrt{ga} = d\sqrt{\frac{2g}{a}}</math></p> <p><math>d = \frac{a}{2\sqrt{2}} = a\frac{\sqrt{2}}{4} = 0.35a \text{ (awrt)}</math></p> <p>(d) Partly under gravity, partly SHM</p>	<p>M1</p> <p>A1 (2)</p> <p>M1 M1 A1</p> <p>M1 A1</p> <p>M1 A1 (7)</p> <p>M1</p> <p>A1 ft on <math>\omega</math></p> <p>A1 (3)</p> <p>B1 B1 (2)</p> <p><b>(14 marks)</b></p>
7.	<p>(a) <math>\frac{1}{2}mu^2 = mgl(1 - \cos \theta)</math></p> <p><math>u = \sqrt{\frac{2}{3}}gl</math></p> <p>(b) <math>T - mg \cos \theta = \frac{mv^2}{l}</math></p> <p><math>\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgl(1 - \cos \theta)</math></p> <p>eliminating <math>v^2</math>, <math>T = \frac{mg}{3}(9 \cos \theta - 4) \quad (*)</math></p> <p>(c) <math>\max T, \theta = 0, T_{MAX} = \frac{5mg}{3}</math></p> <p><math>\min T, \cos \theta = \frac{2}{3}, T_{MIN} = \frac{2mg}{3}</math></p> <p><math>\frac{2mg}{3} \leq T \leq \frac{5mg}{3}</math></p>	<p>M1 A1 A1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1, A1 cso (6)</p> <p>M1</p> <p>M1 A1</p> <p>A1 (4)</p> <p><b>(14 marks)</b></p>

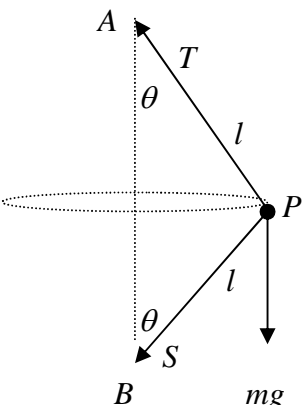
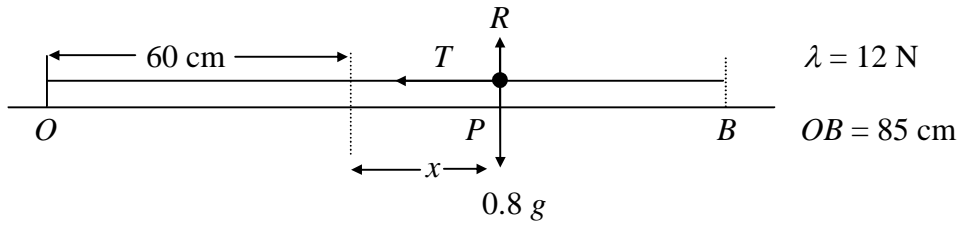
Question Number	Scheme			Marks														
1.		$T_1 = \frac{175 \times 0.2}{1}$ $\frac{175 \times 0.2}{1} + \frac{\lambda \times 0.3}{0.9} = 49$ $\Rightarrow \lambda = 42$	B1  M1 A 1  M1 A1 (5) <b>(5 marks)</b>															
2.	(a) 	3, 4, 5 Δ $R(\uparrow) T \sin \theta = mg$ $T = \frac{5mg}{4}$	B1 M1 A1 (3)															
	(b) R (←)	$T + T \cos \theta = \frac{mv^2}{3l}$ $\frac{8}{5} \times \frac{5mg}{4} = \frac{mv^2}{3l}$ $v = \sqrt{6gl}$	M1 A2  M1 A1 (5)															
	(c)	Could not assume tensions same	B1 (1) <b>(9 marks)</b>															
3.	(a)	<table><tr><td>Cylinder</td><td>half-sphere</td><td>toy</td></tr><tr><td><math>\pi r^2 h \rho</math></td><td><math>\frac{2}{3} \pi r^3 \rho</math></td><td><math>\pi r^2 h \rho + \frac{2}{3} \pi r^3 \rho</math></td></tr><tr><td><math>\frac{h}{2} + r</math></td><td><math>\frac{5r}{8}</math></td><td><math>d</math></td></tr><tr><td colspan="3"><math>\pi r^2 h \rho (\frac{h}{2} + r) + 4 \pi r^3 \rho \frac{5r}{8} = (\pi r^2 h \rho + 4 \pi r^3 \rho) d</math></td></tr><tr><td colspan="3"><math>\Rightarrow d = \frac{h^2 + 2rh + 5r^2}{2(h + 4r)} \quad (*)</math></td></tr></table>	Cylinder	half-sphere	toy	$\pi r^2 h \rho$	$\frac{2}{3} \pi r^3 \rho$	$\pi r^2 h \rho + \frac{2}{3} \pi r^3 \rho$	$\frac{h}{2} + r$	$\frac{5r}{8}$	$d$	$\pi r^2 h \rho (\frac{h}{2} + r) + 4 \pi r^3 \rho \frac{5r}{8} = (\pi r^2 h \rho + 4 \pi r^3 \rho) d$			$\Rightarrow d = \frac{h^2 + 2rh + 5r^2}{2(h + 4r)} \quad (*)$			M1 A1  B1 B1  M1 A1  A1 (7)
Cylinder	half-sphere	toy																
$\pi r^2 h \rho$	$\frac{2}{3} \pi r^3 \rho$	$\pi r^2 h \rho + \frac{2}{3} \pi r^3 \rho$																
$\frac{h}{2} + r$	$\frac{5r}{8}$	$d$																
$\pi r^2 h \rho (\frac{h}{2} + r) + 4 \pi r^3 \rho \frac{5r}{8} = (\pi r^2 h \rho + 4 \pi r^3 \rho) d$																		
$\Rightarrow d = \frac{h^2 + 2rh + 5r^2}{2(h + 4r)} \quad (*)$																		
	(b)	$d = r, \Rightarrow h^2 + 2rh + 5r^2 = 2r(h + 4r)$ $h = \sqrt{3}r$	M1, M1 A1 (3) <b>(10 marks)</b>															

Question Number	Scheme	Marks
4.	<p>(a) <math>\frac{2\pi}{\omega} = \pi \Rightarrow \omega = 2</math></p> <p><math>2.4^2 = 4(a^2 - 0.5^2)</math></p> <p><math>a = 1.3 \text{ m}</math></p> <p>(b) <math>v_{\max} = a\omega = 2.6 \text{ m s}^{-1}</math></p> <p>(c) <math>a\omega^2 = 5.2 \text{ m s}^{-2}</math></p> <p>(d) <math>0.5 = 1.3 \sin 2t</math></p> <p><math>t = \frac{1}{2} \sin^{-1} \left( \frac{0.5}{1.3} \right)</math></p> <p><math>\therefore \text{Total time} = 4t = 0.79 \text{ (2 dp)}</math></p>	<p>B1</p> <p>M1 A1ft</p> <p>A1 (4)</p> <p>B1 (1)</p> <p>B1ft (1)</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1 (5)</p> <p><b>(11 marks)</b></p>
5.	<p>(a) <math>800 \frac{dv}{dt} = \frac{48000}{(t+2)^2}</math></p> <p><math>v = 60 \int \frac{dt}{(t+2)^2} = \frac{-60}{(t+2)} (+c)</math></p> <p><math>t = 0, v = 0 \Rightarrow c = 30</math></p> <p><math>v = 30 - \frac{60}{(t+2)} \Rightarrow v \rightarrow 30 \text{ as } t \rightarrow \infty</math></p> <p>(b) <math>s = \int v dt = 30t - 60 \ln(t+2) (+c)</math></p> <p>substitute in <math>t = 0</math> and <math>t = 6</math></p> <p><math>s = 180 - 60 \ln 8, - -60 \ln 2</math></p> <p><math>\approx 96.8 \text{ m}</math></p>	<p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>A1 (6)</p> <p>M1 A1</p> <p>M1</p> <p>A1, A1</p> <p>A1 (6)</p> <p><b>(12 marks)</b></p>

Question Number	Scheme	Marks
6.	<p>(a) <math>\frac{1}{2} \times \frac{58.8}{4} x^2 = 0.5 \times 9.8 (x + 4)</math></p> <p><math>3x^2 - 2x - 8 = 0</math></p> <p><math>(3x + 4)(x - 2) = 0, \quad x = 2</math></p> <p>Distance fallen = 6 m</p> <p>(b) <math>\frac{1}{2} \times 0.5v^2 = \frac{1}{2} \times \frac{58.8}{4} \times 3^2 - 0.5 \times 9.8 \times 3</math></p> <p><math>v = 14.3 \text{ m s}^{-1}</math></p>	<p>M1 A1 A1</p> <p>M1 A1</p> <p>M1 A1 (7)</p> <p>M1 A1 A1</p> <p>M1 A1 (5)</p> <p><b>(12 marks)</b></p>
7.	<p>(a) <math>\frac{1}{2} mu^2 - \frac{1}{2} mv^2 = mga (1 + \cos 60^\circ)</math></p> <p><math>v^2 = u^2 - 3ga</math></p> <p>(b) <math>R + mg \cos 60^\circ = \frac{mv^2}{a}</math></p> <p><math>R = \frac{m}{a} (6ga - 3ga) - \frac{mg}{2}</math></p> <p><math>= \frac{5mg}{2}</math></p> <p>(c) <math>R = 0 \text{ at } B \Rightarrow \frac{mg}{2} = \frac{mv^2}{a} \Rightarrow v^2 = \frac{1}{2} ag</math></p> <p><math>\Rightarrow u^2 = \frac{7ga}{2} \Rightarrow u = \sqrt{\frac{7ga}{2}}</math></p> <p>(d) <math>(\rightarrow) B \text{ to } C: v \cos 60^\circ \times t = a\sqrt{3}</math></p> <p><math>t = \frac{2a\sqrt{3}}{v}</math></p> <p><math>(\uparrow) B \text{ to } C: 0 = v \sin 60^\circ t - \frac{1}{2} gt^2</math></p> <p><math>\Rightarrow t = \frac{2v \sin 60^\circ}{g} = \frac{v\sqrt{3}}{g}</math></p> <p><math>\therefore \frac{2a\sqrt{3}}{v} = \frac{v\sqrt{3}}{g} \Rightarrow v^2 = 2ga</math></p> <p><math>\Rightarrow u^2 = 5ga</math></p> <p><math>\Rightarrow u = \sqrt{5ga}</math></p> 	<p>M1 A1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>A1 (3)</p> <p>M1</p> <p>M1 A1 (3)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>A1 (7)</p> <p><b>(16 marks)</b></p>

Question Number	Scheme	Marks
1.	 <p>Attempt to relate <math>Fd</math> to EPE</p> $\frac{2}{3} mg d = \frac{4mg\left(\frac{a}{2}\right)^2}{2a}$ <p>Final answer: <math>d = \frac{3}{4} a</math></p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>M1 A1 ft</p> <p>A1 (6)</p> <p><b>(6 marks)</b></p>
2.	 <p>(<math>\uparrow</math>) <math>R \cos 10^\circ = mg</math></p> <p>(<math>\leftrightarrow</math>) <math>R \sin 10^\circ = \frac{mv^2}{r}</math></p> <p>Solving for <math>r</math>: <math>r = \left[ \frac{18^2}{g \tan 10^\circ} \right]</math></p> <p><math>r = 190</math> (m) [Accept 187, 188]</p>	<p>M1 A1</p> <p>M1 A1 ft</p> <p>M1</p> <p>A1 (6)</p> <p><b>(6 marks)</b></p>
3.	<p>(a) <math>\frac{1}{10} x(4 - 3x) = 0.2 a</math></p> <p><math>\frac{1}{10} x(4 - 3x) = 0.2v \frac{dv}{dx}</math> or <math>\frac{1}{10} x(4 - 3x) = 0.2 \frac{d(\frac{1}{2} v^2)}{dx}</math></p> <p>Integrating : <math>v^2 = 2x^2 - x^3 (+ C)</math> or equivalent</p> <p>Substituting <math>x = 6, v = 0</math> to find candidate's <math>C</math></p> <p><math>v^2 = 2x^2 - x^3 + 144</math></p> <p>(b) Substituting <math>x = 0</math> and finding <math>v</math>; <math>v = 12</math> (m s<sup>-1</sup>)</p>	<p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p> <p>M1; A1 ft (2)</p> <p><b>(9 marks)</b></p>

(ft = follow through mark)

Question Number	Scheme	Marks
4. (a)	 $(\updownarrow) (T - S) \cos \theta = mg$ $(\leftrightarrow) (T + S) \sin \theta = mr\omega^2$ $= m(l \sin \theta) \omega^2$ <p>Finding <math>T</math> in terms of <math>l, m, \omega^2</math> and <math>g</math></p> $T = \frac{1}{6} m(3l\omega^2 + 4g) \quad (*)$	<p>M1 A1</p> <p>M1 A1 ft</p> <p>A1</p> <p>M1</p> <p>A1 (7)</p>
(b)	$S = \frac{1}{6} m(3l\omega^2 - 4g)$ <p>any correct form</p>	M1 A1 (2)
(c)	<p>Setting <math>S \geq 0</math>; <math>\omega^2 \geq \frac{4g}{3l} \quad (*)</math></p> <p>(no wrong working seen)</p>	M1 A1 (2)
		(11 marks)
5. (a)	 <p>Hooke's Law: <math>T = \frac{12x}{0.6} [= 20x]</math></p> <p>Equation of motion: <math>(-)T = 0.8 \ddot{x}</math></p> $-\frac{12x}{0.6} = 0.8 \ddot{x} \quad \ddot{x} = -25x$ <p>Finding <math>\omega</math> from derived equation of form <math>\ddot{x} = -\omega^2 x</math></p> <p>Period = <math>\frac{2\pi}{\omega} = \frac{2\pi}{5} \quad (*)</math> <span style="float: right;">no incorrect working seen</span></p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (5)</p>
(b)	<p>Substituting (candidate's) <math>\omega</math> and <math>a</math> in <math>\omega^2 a = 25 \times 0.25 = 6.25 \text{ (m s}^{-2}\text{)}</math></p> <p>(or finding <math>T_{\max} = 0.8a \Rightarrow a = 5/0.8 = 6.25</math>)</p>	M1; A1 (2)
(c)	<p>Complete method for <math>x</math>; <math>x = 0.25 \cos 10^\circ \text{ } (-0.2098)</math></p> <p>Using <math>v^2 = \omega^2 (a^2 - x^2) \Rightarrow v = (\pm) 5 \sqrt{[(0.25)^2 - (0.25 \cos 10^\circ)^2]}</math></p> $v = (\pm) 0.68 \text{ (m s}^{-1}\text{)}$	<p>M1 A1</p> <p>M1 A1 ft</p> <p>A1 (5)</p>
(d)	<p>Direction <math>\overrightarrow{OB}</math> or equivalent</p>	B1 (1)
		(13 marks)

(ft = follow through mark; (\*) indicates final line is given on the paper)



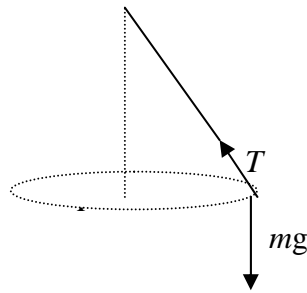
Question Number	Scheme	Marks
6.	(a) Energy: $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga(1 - \cos \theta)$	M1 A1 A1
	Radial: $(\pm R) + mg \cos \theta = \frac{mv^2}{a}$	M1 A1
	Eliminating $v$ and finding $\cos \theta = \frac{u^2 + 2ga}{3ga}$	M1, A1 (7)
	(b) Energy (C and ground): $\frac{1}{2}m\left(\frac{9ag}{2}\right) - \frac{1}{2}mv^2 = mga(1 - \cos \theta)$	M1 A1
	Eliminating $v$ : $\frac{1}{2}m\left(\frac{9ag}{2}\right) - \frac{1}{2}mag \cos \theta = mga(1 + \cos \theta)$ $\cos \theta = \frac{5}{6}$ $\theta = 34^\circ$	M1 A1 M1 A1 ft A1 (7) <b>(14 marks)</b>
Alt (b)	Or energy (A and ground): $\frac{1}{2}m\left(\frac{9ag}{2}\right) - \frac{1}{2}mu^2 = 2mga$ $u^2 = \frac{1}{2}ga$ Using with (a) to find $\cos \theta = \frac{5}{6}$ ; $\theta = 34^\circ$	M1 A1 M1 A1 M1 A1; A1 (7)
Alt	Projectile approach: $V_x = v \cos \theta$ ; $V_y^2 = (v \sin \theta)^2 + 2ga(1 + \cos \theta)$ $\left(\frac{9ag}{2}\right) = V_x^2 + V_y^2 \Rightarrow \left(\frac{9ag}{2}\right) - v^2 = 2ga(1 + \cos \theta)$ – M1 A1, then scheme	

(ft = follow through mark)

Question Number	Scheme	Marks
7.	(a) $V = \pi \int y^2 \, dx = \frac{1}{4} \pi \int (x-2)^4 \, dx$	M1
	$\int (x-2)^4 \, dx = \frac{1}{5} (x-2)^5$	M1 A1
	$V = \frac{8\pi}{5}$	A1 (4)
	(b) Using $\pi \int xy^2 \, dx = \frac{1}{4} \pi \int x(x-2)^4 \, dx$	M1
	Correct strategy to integrate [e.g. substitution, expand, by parts]	M1
	[e.g. $\frac{1}{4} \pi \int (u-2)^4 \, du$ ; $\frac{1}{4} \pi \int (x^5 - 8x^4 + 24x^3 - 32x^2 + 16x) \, dx$ ]	
	$= \frac{1}{4} \pi \left[ \frac{2u^5}{5} + \frac{u^6}{6} \right]$ or $\frac{1}{4} \pi \left[ \frac{x^6}{6} - \frac{8x^5}{5} + 6x^4 - \frac{32x^3}{3} + 8x^2 \right]$	M1 A1
	$= \frac{8\pi}{15}$ limits need to be used correctly	A1 (7)
	$V_c(\rho) \bar{x} = \pi(\rho) \int xy^2 \, dx$ seen anywhere	M1
	$\bar{x} = \frac{1}{3} \text{ cm } (*)$ no incorrect working seen	A1
(c)	Moments about B: $8A = 10W - 2W(\frac{1}{3})$	M1 A1 A1
	$A = \frac{59W}{12}$ (4.9W)	M1 A1 (5)
		<b>(16 marks)</b>

(ft = follow through mark; (\*) indicates final line is given on the paper)

1.



(a) ( $\Downarrow$ )  $T \cos 60^\circ = mg \Rightarrow T = 2mg$  \* B1 (1)

(b) ( $\leftrightarrow$ )  $T \sin 60^\circ = mr\omega^2$  M1A1  
[Omission of  $m$  is M0]

Attempt at  $r = L \sin 60^\circ$  M1

( $T \sin 60^\circ = m L \sin 60^\circ \omega^2$ )

$\omega = \sqrt{\frac{2g}{L}}$  A1 (4)

(c) Applying Hooke's Law:  $2mg = \frac{\lambda}{(\frac{3}{5}L)} (L - \frac{2}{5}L); \quad \lambda = 3mg$  M1;A1 (2)

[ $L$  in denominator is M0]

[7]

2.

(a) Integration of  $-4e^{-2t}$  w.r.t.  $t$  to give  $v = 2e^{-2t}$  (+c) B1

Using initial conditions to find  $c$  ( $-1$ ) or  $v - 1 = [f(t)]_0^t$  M1

$v = 2e^{-2t} - 1 \text{ ms}^{-1}$  A1 (3)

(b) **Integrating**  $v$  w.r.t.  $t$ ;  $x = -e^{-2t} - t$  (+c) M1;A1✓

Using  $t=0, x=0$  **and** finding value for  $c$  ( $c=1$ ) M1

Finding  $t$  when  $v=0$ ;  $t = \frac{1}{2} \ln 2$  or equiv., 0.347 M1;A1✓

[both f.t. marks dependent on  $v$  of form  $ae^{-2t} \pm b$ ]

$x = \frac{1}{2} (1 - \ln 2) \text{ m}$  or 0.153 m (awrt) A1 (6)

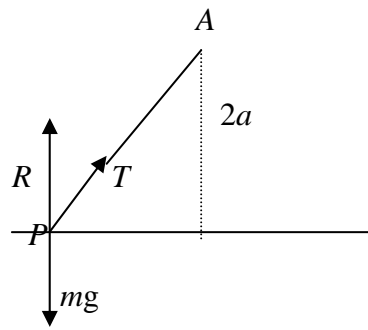
[9]

[For A1, exact form must be two termed answer]

3. (a)	$F = \frac{k}{x^2}$ [k may be seen as $Gm_1m_2$ , for example]	M1
	Equating $F$ to $mg$ at $x = R$ , [ $mg = \frac{k}{R^2}$ ]	M1
	Convincing completion [ $k = mgR^2$ ] to give $F = \frac{mgR^2}{x^2}$ *	A1 (3)
	[Note: $r$ may be used instead of $x$ throughout, then $r \rightarrow x$ at end.]	
(b)	Equation of motion: $(m)a = (-) \frac{(m)gR^2}{x^2}$ ; $(m)v \frac{dv}{dx} = - \frac{(m)gR^2}{x^2}$	M1;M1
	Integrating: $\frac{1}{2} v^2 = \frac{gR^2}{x}$ (+ c) or equivalent	M1A1
	[S.C: Allow A1✓ if A0 earlier due to “+” only]	
	Use of $v^2 = \frac{3gR}{2}$ , $x = R$ to find $c$ [ $c = -\frac{1}{4}gR$ ] or use in def. int.	M1
	[Using $x = 0$ is M0] $[v^2 = \frac{2gR^2}{x} - \frac{gR}{2}]$	
	Substituting $x = 3R$ and finding $V$ ; $V = \sqrt{\frac{gR}{6}}$	M1;A1 (7)
	[Using $x = 2R$ is M0]	
	Alternative in (b)	
	Work/energy $(-) \int_R^a \frac{mgR^2}{x^2} dx ; = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	M1;M1
	Integrating: $[\frac{mgR^2}{x} - \frac{mgR^2}{R}] = \frac{1}{2}mv^2 - \frac{1}{2}m \frac{3gR}{2}$	M1A1M1
	Final 2 marks as scheme	M1A1
	[Conservation of energy scores 0]	

[10]

4.



(a) Length of string =  $\frac{10}{3}a$

B1

$$\text{EPE} = \frac{\frac{1}{2}mg}{2a} (L - a)^2$$

M1

$$= \frac{49}{36} mga$$

A1 (3)

(b) Energy equation:  $\frac{1}{2}mv^2 + \frac{\frac{1}{2}mg}{2a} a^2 = (\frac{49}{36} mga)_C$

M1A1☆

$$v = \frac{2}{3} \sqrt{5ga} \text{ or equivalent}$$

A1 (3)

(c) When string at angle  $\theta$  to horizontal, length of string =  $\frac{2a}{\sin \theta}$

$$\Rightarrow \text{Vert. Comp. of } T, T_V = T \sin \theta = \frac{mg}{2a} \left( \frac{2a}{\sin \theta} - a \right) \sin \theta$$

$$= \frac{mg}{2} (2 - \sin \theta)$$

M1A1

(⇕)  $R + T_V = mg$  and find  $R = \dots$

M1

$$R = mg - \frac{mg}{2} (2 - \sin \theta) = \frac{mg}{2} \sin \theta$$

A1

$$\Rightarrow R > 0 \text{ (as } \sin \theta > 0), \text{ so stays on table}$$

A1 (5)

[Alternative final 3 marks: As  $\theta$  increases so  $T_V$  decreases M1

Initial  $T_V$  (string at  $\beta$  to hor.) =  $\frac{7}{10}mg$  A1

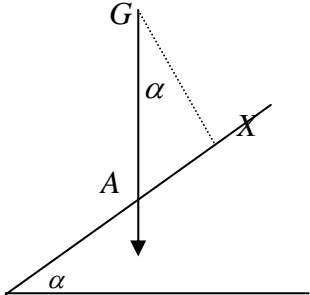
$$\Rightarrow T_V \leq \frac{7}{10}mg < mg, \text{ so stays on table A1]}$$

[11]



6.	(a)	Cylinder	Hemisphere	S	
		Masses	$(\rho)\pi(2a)^2(\frac{3}{2}a)$ [6 $\pi a^3$ ] [18]	$(\rho)\frac{2}{3}\pi a^3$ [2]	$(\rho)(\frac{16}{3}\pi a^3)$ [16]
					M1A1
		Distance of CM from O	$\frac{1}{8}a$	$\frac{3}{8}a$	$\bar{x}$
					B1B1
		Moments equation:	$6\pi a^3(\frac{3}{4}a) - \frac{2}{3}\pi a^3(\frac{3}{8}a) = \frac{16}{3}\pi a^3 \bar{x}$		
			$\bar{x} = \frac{51}{64}a$		
			A1 (6)		
			M1		

(b)



$G$  above “A” seen or implied  
or  $mg \sin \alpha (GX) = mg \cos \alpha (AX)$

$\tan \alpha = \frac{AX}{XG} = \frac{2a}{\frac{3}{2}a - \bar{x}}$

$[GX = \frac{3}{2}a - \frac{51}{64}a = \frac{45}{64}a, \tan \alpha = \frac{128}{45}] \quad \alpha = 70.6^\circ$

A1 (3)

(c) Finding  $F$  and  $R$  :  $R = mg \cos \beta$ ,  $F = mg \sin \beta$  M1

Using  $F = \mu R$  and finding  $\tan \beta [= 0.8]$  M1

$$\beta = 38.7^\circ \quad \text{A1 (3)}$$

[12]

7.	(a) Energy: $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga \sin \theta$	M1
	$v^2 = \frac{3}{2}ga + 2ga \sin \theta$	A1 (2)
	(b) Radial equation: $T - mg \sin \theta = m \frac{v^2}{a}$	M1A1
	$T = \frac{3mg}{2}(1 + 2\sin \theta) \text{ any form}$	A1☆ (3)
	(c) Setting $T = 0$ and solving trig. equation; $(\sin \theta = -\frac{1}{2}) \Rightarrow \theta = 210^\circ *$	M1;A1(2)
	(d) Setting $v = 0$ in (a) and solving for $\theta$	M1
	$\sin \theta = -\frac{3}{4}$ so not complete circle	A1 (2)
	OR Substituting $\theta = 270^\circ$ in (a); $v^2 < 0$ so not possible to complete	
	(e) No change in PE $\Rightarrow$ no change in KE (Cof E) so $v = u$	B1 (1)
	(f) When string becomes slack, $V^2 = \frac{1}{2}ga$ [ $\sin \theta = -\frac{1}{2}$ in (a)]	B1☆
	Using fact that horizontal component of velocity is unchanged	M1
	$\sqrt{\frac{ga}{2}} \cos 60^\circ = \sqrt{\frac{3ga}{2}} \cos \phi$	
	$\cos \phi = \sqrt{\frac{1}{12}} \Rightarrow \phi = 73.2^\circ$	M1A1 (4)
		[14]



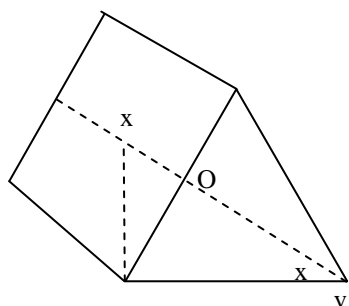
Question Number	Scheme	Marks
1.	$1000 \text{ r.p.m} = \frac{1000 \times 2\pi}{60} \text{ rad/s}$ $v = 0.035 \times \frac{1000 \times 2\pi}{60} = 3.67 \text{ ms}^{-1} \text{ (3 SF)}$	B1 M1 A1 <b>(3 marks)</b>
2.	$\text{Extn at bottom} = \frac{a}{\cos \alpha} - a = \frac{2a}{3} \text{ (0.67a or better)}$ $\text{Energy: } mga \tan \alpha = \frac{2\lambda \left(\frac{2a}{3}\right)^2}{2a}$ $3mg = \lambda$	M1 A1 M1 A1 A1 ft M1 A1 Second M0 if treated as equilibrium Third M1 for solving for $\lambda$ <b>(7 marks)</b>
3. (a)	$mg \sin 30^\circ - mx^2 = ma$ $\frac{g}{2} - x^2 = v \frac{dv}{dx} \text{ or } \frac{d\left(\frac{1}{2}v^2\right)}{dx}$ $\frac{gx}{2} - \frac{x^3}{3} (+C) = \frac{v^2}{2}$ $x = 2 : g - \frac{8}{3} = \frac{v^2}{2}$ $v = 3.8 \text{ ms}^{-1} \text{ (3.78)}$	M1 A1 M1 M1 A1 M1 A1 (7) Third M1 for attempting to integrate
(b)	$v = 0: \frac{gx}{2} - \frac{x^3}{3} = 0$ $x^2 = \frac{3g}{2} \Rightarrow x = 3.8, (3.83), \sqrt{\frac{3g}{2}}$	M1 M1 A1 c.s.o must have integrated for first M1 <b>(10 marks)</b>

(ft = follow through mark)

Question Number	Scheme	Marks
4. (a)	$(\uparrow), R = mg$ $m \frac{4a}{3} \omega^2$ (seen and used)	B1 B1
	$m \frac{4a}{3} \omega^2 \leq \frac{3}{5} mg$	M1
	$\omega^2 \leq \frac{9g}{20a} *$	A1 c.s.o (4)
(b)	$T = \frac{2mg}{a} \frac{a}{3} = \frac{2mg}{3}$	B1
	$(\rightarrow), \frac{3}{5}mg + \frac{2mg}{3} \geq m \frac{4a}{3} \omega_{\max}^2$	M1 A1 f.t
	$\frac{19g}{20a} = \omega_{\max}^2$	A1
	$(\rightarrow), -\frac{3}{5}mg + \frac{2mg}{3} \leq m \frac{4a}{3} \omega_{\min}^2$	M1 A1 f.t
	$\frac{g}{20a} = \omega_{\min}^2$	A1 (7)
	If only one answer, must be clear whether max or min for final A1	(11 marks)

(ft = follow through mark; (\*) indicates final line is given on the paper)

Question Number	Scheme				Marks
5. (a)	<div> <div>Cylinder (<math>36\pi r^3</math>)</div> <div>Cone (<math>12\pi r^3</math>)</div> <div>Toy (<math>48\pi r^3</math>)</div> </div>				
	mass ratio	3	1	4	B1
	dist. From $O$	$2r$	$(- )r$	$\bar{x}$	B1
	$(3 \times 2r) - r = 4\bar{x}$ $\frac{5r}{4} = \bar{x}$				M1 A1 A1 (5)
	<p>M1 for clear attempt at <math>\Sigma mx = \bar{x} \Sigma m</math> – correct no. of terms.            If distances not measured from <math>O</math>, B1B1M1A1 available.            AG vertical, seen or implied</p>				M1
(b)	$\tan \theta = \frac{3r}{4r - \bar{x}}$ $\theta = 47.5^\circ \text{ (1 d.p.)}$				M1 A1 A1 (4) second M1 for use of tan
(c)	<p>Sim <math>\Delta</math>'s: <math>\frac{OX}{3r} = \frac{3r}{4r} (= \tan \alpha)</math></p>				M1
	$\Rightarrow OX = \frac{9r}{4}$				A1
	$\bar{x} < OX$				M1
	$\Rightarrow \text{won't topple}$ <p>Note that second M1 is independent, for the general idea.</p>				A1 c.s.o (4) <b>(13 marks)</b>



(ft = follow through mark; (\*) indicates final line is given on the paper)

Question Number	Scheme	Marks
6.	All M marks require correct number of terms with appropriate terms resolved $B \text{ to } C: \frac{1}{2}mv^2 - \frac{1}{2}m20^2 = mg \times 50(1 - \sin 30^\circ)$ $v = 30 \text{ ms}^{-1} (29.8)$	M1 A1 A1 (3)
	$(\uparrow) \text{ at } C, \quad R - mg = m \frac{890}{50}$ $R = 1900 \text{ N} (1930 \text{ N})$	M1 A1 ft A1 (3)
	$C \text{ to } D: \frac{1}{2}m890 - \frac{1}{2}mw^2 = mg \times 50(1 - \cos 30^\circ)$ $w = 28 \text{ ms}^{-1} (27.5)$	M1 A1 ft A1 (3)
	Before: $R = mg \cos \theta$ After: $R = mg \cos \theta + m \frac{20^2}{50}$ Change = $70 \times \frac{20^2}{50} = 560 \text{ N}$	B1 M1 A1 A1 c.s.o (4)
	$(e) \quad \text{Lower speed at } C \Rightarrow R \text{ reduced}$	M1 A1 (2) <b>(15 marks)</b>

(ft = follow through mark)

Question Number	Scheme	Marks
7. (a)	$(-) \frac{21.6x}{2} = 0.3 \ddot{x}$ $-36x = \ddot{x}$ $\text{S.H.M., period} = \frac{2\pi}{\sqrt{36}} = \frac{\pi}{3} *$	M1 A1 M1 A1 c.s.o. (4)
(b)	At A: $v = aw = 1.5 \times 6 = 9 \text{ ms}^{-1}$	M1 A1 (2)
(c)	$x = a \cos \omega t$ $0.75 = 1.5 \cos 6t$ $\frac{\pi}{3} = 6t \Rightarrow t = \frac{\pi}{18} \text{ (no decimals)}$	M1 M1 A1 (3)
(d)	$(-) \frac{21.6x}{2} = 0.5 \ddot{x}$ $-21.6x = \ddot{x} \Rightarrow \text{S.H.M., } \omega = \sqrt{21.6}$ $\text{At collision: CLM: } 0.3 \times 9 = 0.5v \Rightarrow v = 5.4$ $a \times \sqrt{21.6} = 5.4$ $a = 1.16 \text{ m (3SF)}$	M1 A1 A1 M1 A1 ft M1 A1 (7) <b>(16 marks)</b>

(ft = follow through mark; (\*) indicates final line is given on the paper)

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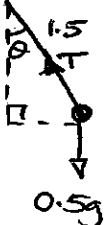
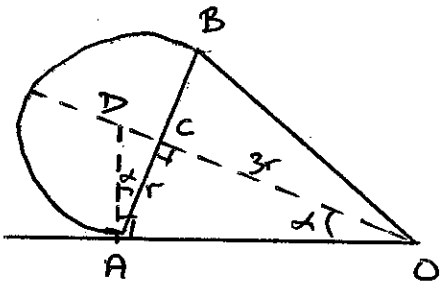
January 2005

Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: **Mechanics**

Paper: **M3**

Question Number	Scheme	Marks
1.(a)	 $r = 1.5 \sin \theta$ $T \sin \theta = m r \omega^2$ $T \sin \theta = 0.5 \times 1.5 \sin \theta \times 2.7^2$ $T = \underline{5.4675 \text{ N}}$	B1 M1 A1 A1 (4)
(b)	$T \cos \theta = 0.5g$ $\cos \theta = \frac{0.5g}{5.4675}$ $\theta = \underline{26^\circ} \text{ (nearest degree)}$	M1 A1 A1 (3) (7)
2.(a)	$\frac{3r}{4} ; \frac{3r}{8}$ $-m \cdot \frac{3r}{4} + M \cdot \frac{3r}{8} = (m+M) \pi$ $\frac{3r(M-2m)}{8(M+m)} = \pi$	B1 ; B1 M1 A1 A1 (5)
(b)	 $CD = r \tan \alpha$ $= r \times \left( \frac{r}{3r} \right)$ $= \frac{1}{3} r$ <p>No equil<sup>n</sup> <math>\Rightarrow \bar{x} &gt; CD</math></p> $\frac{3r(M-2m)}{8(M+m)} > \frac{r}{3}$ $9(M-2m) > 8(M+m)$ $M > 26m$	M1 A1  M1 A1 (4) (9)

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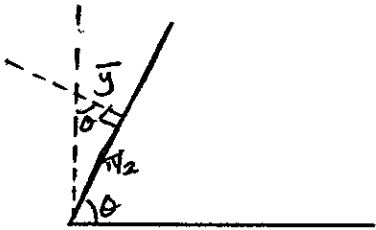
January 2005

Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: **Mechanics**

Paper: **M3**

Question Number	Scheme	Marks
3.(c)	$\int_0^{\pi} \frac{1}{2} y^2 dx = \int_0^{\pi} \frac{1}{2} \sin^2 x dx$ $= \frac{1}{4} \int_0^{\pi} (1 - \cos 2x) dx$ $= \frac{1}{4} \left[ x - \frac{1}{2} \sin 2x \right]_0^{\pi}$ $= \frac{\pi}{4}$ $\bar{y} = \frac{\frac{\pi}{4}}{\int_0^{\pi} \sin x dx} = \frac{\frac{\pi}{4}}{\frac{2}{2}} = \frac{\pi}{8}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1 (6)</p>
(b)	 $\tan \theta = \frac{\pi/2}{y}$ $= 4$ $\theta = 75.96^\circ$	<p>M1</p> <p>A1 ✓</p> <p>A1 (3)</p> <p>(9)</p>

# EDEXCEL

190 High Holborn London WC1V 7BH

January 2005

Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: **Mechanics**

Paper: **M3**

Question Number	Scheme	Marks
4. (a)	$6 = 2\pi\omega \Rightarrow \omega = \pi/3$ $a = 2L$ $x = 2L \cos \omega t$ $2L - b = 2L \cos(\pi/3 \cdot 3/4)$ $\underline{b = L(2 - \sqrt{2})} \quad *$	<p>M1 B1  M1 A1 ✓  A1 c.s.o. (5)</p>
(b)	$\ddot{x} = -2L\omega \sin \omega t$ $= -2L \frac{\pi}{3} \sin \frac{\pi}{4}$ $\text{Speed} = \frac{\sqrt{2}L\pi}{3}$	<p>M1  A1 (2)</p>
(c)	$\frac{1}{2}(2 - \sqrt{2})L = 2L \sin \omega t$ $t = 0.1469 \dots$ $\therefore \text{Total time} = 2 \times 0.14$ $= \underline{0.28 \text{ (2dp)}}$	<p>M1 A1  A1 (3) (10)</p>
5. (a)	$\frac{dv}{dt} = -\frac{3}{\sqrt{t+4}}$ $v = -3 \int (t+4)^{-\frac{1}{2}} dt$ $v = -6(t+4)^{\frac{1}{2}} + C$ $t=0, v=18: 18 = -6 \times 2 + C \Rightarrow C = 30$ $\underline{v = 30 - 6\sqrt{t+4}} \quad *$	<p>M1  M1 A1 M1 A1 c.s.o. (5)</p>
(b)	$x = \int 30 - 6(t+4)^{\frac{1}{2}} dt$ $= 30t - 4(t+4)^{\frac{3}{2}} + D$ $t=0, x=0: 0 = 0 - 4 \times 8 + D \Rightarrow D = 32$ $v=0 \Rightarrow 30 - 6\sqrt{t+4} = 0 \Rightarrow t = 21$ $\text{When } t=21, x = 30 \times 21 - 4 \times 5^2 + 32$ $= \underline{\underline{162 \text{ (2)}}}$	<p>M1 A1 M1 M1 A1 M1 A1 (7) (12)</p>



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January 2005

Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: **Mechanics**

Paper: **M3**

Question Number	Scheme	Marks
6.(a)	$KE_{\text{loss}} + PE_{\text{loss}} = EPE_{\text{Gau.}}$ $\frac{1}{2} \cdot m 2g L + m g 3L = \frac{\lambda (3L)^2}{2L}$ $\times \frac{8mg}{9} = \lambda$	<p>M1 A2 (1.e.e.) A1 (4)</p>
(b)	$mg - T = m \ddot{x}$ $mg - \frac{8mg}{9L}(x+e) = m \ddot{x}$ $-\frac{8g}{9L}x = \ddot{x}$ <p>Hence SHM about D</p>	<p>M1 A1 M1 A1 A1 c.s.o. (5)</p>
(c)	<p>(i) Period = <math>\frac{2\pi}{\omega} = 2\pi \sqrt{\frac{9L}{8g}} = 3\pi \sqrt{\frac{L}{2g}}</math></p> <p>(ii) <math>mg = \frac{8mg}{9L}e \Rightarrow e = 9L/8</math></p> <p><math>a = 3L - 9L/8 = 15L/8</math></p> <p><math>v_{\text{max}} = a\omega = \frac{15L}{8} \sqrt{\frac{8g}{9L}}</math></p> <p><math>= \frac{5}{4} \sqrt{2gL}</math></p>	<p>M1 A1 B1 M1 A1 (5) (14)</p>

# EDEXCEL

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January 2005

Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: **Mechanics**

Paper: **M3**

Question Number	Scheme	Marks
7.(a)	$\frac{1}{2}mv^2 - 15 = mg 5(1 - \cos 60^\circ)$ $v = 8 \text{ ms}^{-1}$	M1 A1 A1 A1 (4)
(b)	$\frac{1}{2}mv^2 = mg 5(1 - \cos 60^\circ)$ $v = 7 \text{ ms}^{-1}$ <p><u>Alt:</u> <math>60 \times 8 - 3m = (60 + m) 7</math></p> $480 - 3m = 420 + 7m$ $60 = 10m$ $6 = m$	M1 A1 A1 M1 A1✓ A1✓ A1 (7)
(c)	$T - 66g = \frac{66 \times 7^2}{5}$ $T = 132g$ $= \underline{1290 \text{ (1294) N}}$	M1 A1✓ A1 (3)
		(4)

June 2005

Final Version

**6679 Mechanics M3**  
**Mark Scheme**

The following abbreviations are used in this scheme.

- M     A method mark. These are awarded for 'knowing a method and attempting to apply it'.
- A     An accuracy mark. Can only be awarded if the relevant method mark(s) have been earned.
- B     These marks are independent of method marks.
- cso   correct solution only. There must be no errors in this part of the question to obtain this mark.
- cao   correct answer only.
- ft    follow through. The scheme or marking guidance will specify what is to be followed through.
- oe    or equivalent.
- awrt   answers which round to

[The second mark is dependent on gaining the first mark.

N2L   Newton's second law

LHS   Left hand side of an equation

LM    Linear momentum

RHS   Right hand side of an equation

HL    Hooke's Law.

EPE   Elastic potential energy

→, ↓ etc. Resolving in the appropriate direction

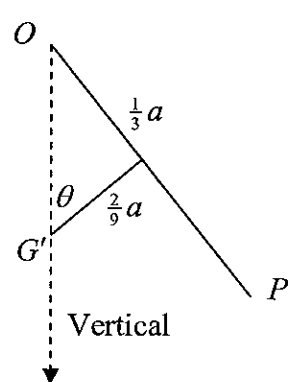
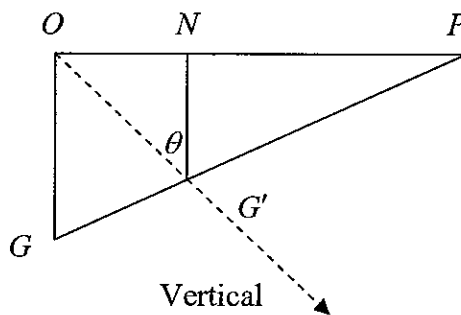
M(A)   Taking moments about A.

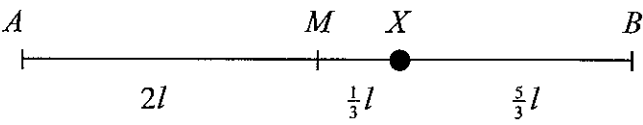
\*     The answer is printed on the paper.

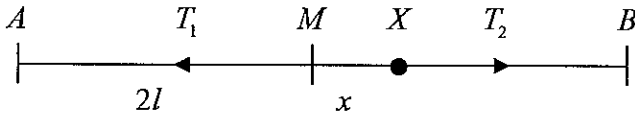
Question Number	Scheme	Marks
1.	<div data-bbox="316 456 711 725"> </div> <div data-bbox="788 456 1315 792"> <p>HL <math>T = \frac{20 \times 0.4}{2} (=4)</math> accept -4</p> <p>[ <math>mg \sin \alpha + T = ma</math></p> <p><math>0.8g \times 0.6 + 4 = 0.8a</math></p> <p><math>a = 10.88 \approx 10.9 \text{ (ms}^{-2}\text{)}</math> accept 11</p> </div>	<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>[6]</p>

Question Number	Scheme	Marks															
2.	<p>(a)</p> <table border="0"> <tr> <td></td> <td>Bowl</td> <td>Lid</td> <td>C</td> <td></td> </tr> <tr> <td>Mass ratio</td> <td>2</td> <td>1</td> <td>3</td> <td>anything in ratio 2 : 1 : 3</td> </tr> <tr> <td><math>\bar{y}</math></td> <td><math>\frac{1}{2}a</math></td> <td>0</td> <td><math>\bar{y}</math></td> <td></td> </tr> </table> <p>M(O) <math>2 \times \frac{1}{2}a = 3\bar{y}</math>  <math>\bar{y} = \frac{1}{3}a</math> *</p> <p>(b)</p> <p>M(A) <math>Mg \times \frac{1}{3}a \sin \theta = \frac{1}{2}Mg \times a \cos \theta</math>  <math>\tan \theta = \frac{3}{2}</math>  <math>\theta \approx 56^\circ</math></p>		Bowl	Lid	C		Mass ratio	2	1	3	anything in ratio 2 : 1 : 3	$\bar{y}$	$\frac{1}{2}a$	0	$\bar{y}$		<p>B1 B1 M1 A1 (4)</p> <p>M1 A1=A1 M1 A1 (5) [9]</p>
	Bowl	Lid	C														
Mass ratio	2	1	3	anything in ratio 2 : 1 : 3													
$\bar{y}$	$\frac{1}{2}a$	0	$\bar{y}$														

Methods involving the location of the combined centre of mass of C and P are considered on the next page.

Question Number	Scheme	Marks																
2.	<p>(b) <i>Methods involving the location of the combined centre of mass of C and P.</i></p> <p><i>G is the centre of mass of C; G' is the combined centre of mass of C and P.</i></p> <p><i>First Alternative</i></p> <table><tr><td></td><td><i>C</i></td><td><i>P</i></td><td><i>C and P</i></td></tr><tr><td>Mass ratios</td><td>2</td><td>1</td><td>3</td></tr><tr><td><math>\bar{y}</math></td><td><math>\frac{1}{3}a</math></td><td>0</td><td><math>\bar{y}</math></td></tr><tr><td><math>\bar{x}</math></td><td>0</td><td><math>a</math></td><td><math>\bar{x}</math></td></tr></table> <p>Finding both coordinates of <math>G'</math></p> $\frac{2}{3}a = 3\bar{y} \Rightarrow \bar{y} = \frac{2}{9}a$ $a = 3\bar{x} \Rightarrow \bar{x} = \frac{1}{3}a$ <div></div> $\tan \theta = \frac{\frac{1}{3}a}{\frac{2}{9}a} = \frac{3}{2}$ $\theta \approx 56^\circ$ <p><i>Second Alternative</i></p> <div></div> <p><math>GG' : G'P = \frac{1}{2}M : M = 1 : 2</math></p> <p><math>OG = \frac{1}{3}a, \quad OP = a</math></p> <p>By similar triangles</p> <p><math>ON = \frac{1}{3}OP = \frac{1}{3}a</math></p> <p><math>NG' = \frac{2}{3}OG = \frac{2}{9}a</math></p> $\tan \theta = \frac{ON}{NG'} = \frac{\frac{1}{3}a}{\frac{2}{9}a} = \frac{3}{2}$ $\theta \approx 56^\circ$		<i>C</i>	<i>P</i>	<i>C and P</i>	Mass ratios	2	1	3	$\bar{y}$	$\frac{1}{3}a$	0	$\bar{y}$	$\bar{x}$	0	$a$	$\bar{x}$	<div><div>M1</div><div>A1</div><div>A1</div></div> <div>M1</div> <div>cao</div> <div>A1</div> <div>(5)</div> <div><div>M1</div><div>A1</div></div> <div>M1</div> <div>cao</div> <div>A1</div> <div>(5)</div>
	<i>C</i>	<i>P</i>	<i>C and P</i>															
Mass ratios	2	1	3															
$\bar{y}$	$\frac{1}{3}a$	0	$\bar{y}$															
$\bar{x}$	0	$a$	$\bar{x}$															

Question Number	Scheme	Marks
3.	<p>(a)</p>  <p>Elastic energy when <math>P</math> is at <math>X</math>: <math>E = \frac{4mg\left(\frac{2}{3}l\right)^2}{2l} + \frac{4mg\left(\frac{4}{3}l\right)^2}{2l} \left( = \frac{40mgl}{9} \right)</math></p> <p><math>\frac{1}{2}mV^2 + 2 \times \frac{4mgl^2}{2l} = \frac{4mg\left(\frac{2}{3}l\right)^2}{2l} + \frac{4mg\left(\frac{4}{3}l\right)^2}{2l}</math></p> <p><math>\frac{1}{2}V^2 + 4gl = \frac{8}{9}gl + \frac{32}{9}gl</math></p> <p><math>V^2 = \frac{8gl}{9}</math> solving for <math>V^2</math></p> <p><math>V = \left(\frac{8gl}{9}\right)^{\frac{1}{2}}</math> or exact equivalents</p> <p>(b) The maximum speed occurs when <math>a = 0</math></p> <p>At <math>M</math> the particle is in equilibrium (the sum of the forces is zero) <math>\Rightarrow a = 0</math></p> <p><i>The alternative method using Newton's Second Law is considered on the next page.</i></p>	<p>M1 A1</p> <p>M1A1=A1ft</p> <p>M1</p> <p>A1 (7)</p> <p>B1</p> <p>B1 (2)</p> <p>[9]</p>

Question Number	Scheme	Marks
3.	<p><i>Alternative using Newton's second law.</i></p> <p>(a)</p>  <p>HL <math>T_1 = \frac{4mg(l+x)}{l}, T_2 = \frac{4mg(l-x)}{l}</math></p> <p>N2L <math>m\ddot{x} = T_2 - T_1 = -\frac{8mg}{l}x</math></p> <p>This is SHM, centre M</p> <p><math>a = \frac{l}{3}, \omega^2 = \frac{8g}{l}</math></p> <p><math>v^2 = \omega^2(a^2 - x^2) \Rightarrow v^2 = \frac{8g}{l}\left(\frac{l^2}{9} - x^2\right)</math> Depends on showing SHM</p> <p>At M, <math>x = 0, V^2 = \frac{8gl}{9}, V = \left(\frac{8gl}{9}\right)^{\frac{1}{2}}</math> or exact equivalents</p> <p>(b) The particle is performing SHM about the mid-point of AB. The maximum speed occurs at the centre of the oscillation (when <math>x = 0</math>)</p>	<p>M1 A1</p> <p>A1, A1ft</p> <p>M1</p> <p>M1, A1 (7)</p> <p>B1 B1 (2) [9]</p>



Question Number	Scheme	Marks
4.	<div data-bbox="300 376 609 721" data-label="Image"> </div> <p data-bbox="236 902 432 976">Note: <math>x = \frac{\sqrt{3}}{2}r</math></p> <div data-bbox="826 297 1252 371">(a) <math>\sin \theta = \frac{\frac{1}{2}r}{r} = \frac{1}{2} \quad (\Rightarrow \theta = 30^\circ)</math></div> <div data-bbox="858 409 1090 495"> <math display="block">\uparrow \quad R \sin \theta = mg</math> <math display="block">R = 2mg</math> </div> <div data-bbox="820 551 1241 645">(b) <math>\rightarrow \quad R \cos \theta = mx\omega^2</math> <math display="block">= m(r \cos \theta)\omega^2</math> </div> <div data-bbox="1013 696 1157 781"> <math display="block">\omega = \left( \frac{2g}{r} \right)^{\frac{1}{2}}</math> </div> <div data-bbox="804 797 1310 891"> <math display="block">T = \frac{2\pi}{\omega} = 2\pi \left( \frac{r}{2g} \right)^{\frac{1}{2}} \text{ or exact equivalent}</math> </div>	<div data-bbox="1342 320 1378 349">B1</div> <div data-bbox="1342 421 1517 499">M1 A1 A1 (4)</div> <div data-bbox="1342 555 1517 633">M1 A1 A1</div> <div data-bbox="1342 723 1378 752">A1</div> <div data-bbox="1342 824 1517 864">M1 A1 (6)</div> <div data-bbox="1465 925 1517 958">[10]</div>

Question Number	Scheme	Marks
5.	<div data-bbox="260 360 770 730"> </div> <p>(a) <math>\frac{1}{2}mv^2 = mg(a \cos \alpha - a \cos \theta)</math>  <math>v^2 = 2ga(\cos \alpha - \cos \theta)</math> * cso</p> <p>(b) <math>[mg \cos \theta (-R) = \frac{mv^2}{a} \quad (R = 0)</math>  <math>g \cos \theta = 2g\left(\frac{3}{4} - \cos \theta\right)</math>  <math>\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} \text{ (accept } 60^\circ)</math></p> <p>(c) From A to B <math>\frac{1}{2}mw^2 = mg(a + a \cos \alpha)</math>  <math>w^2 = 2ga\left(1 + \frac{3}{4}\right) \Rightarrow w = \left(\frac{7ga}{2}\right)^{\frac{1}{2}}</math></p> <p><i>Alternative solutions to 5(c) are considered on the next page.</i></p>	<p>M1 A1 <u>A1</u>  A1 (4)</p> <p>M1 A1=A1  M1  A1 (5)</p> <p>M1 A1 <u>A1</u>  A1 (4)  [13]</p>

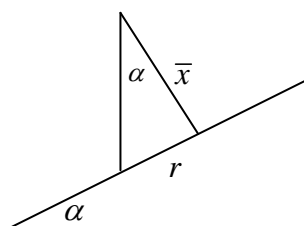
Question Number	Scheme	Marks
5.	<p><i>Alternatives to 5(c)</i></p> <p><i>From P to C</i></p> $v_P^2 = 2ga \left( \frac{3}{4} - \frac{1}{2} \right) = \frac{ga}{2}$ $\frac{1}{2}mw^2 - \frac{1}{2}m \left( \frac{ga}{2} \right) = mg(a + a \cos \theta)$ $w^2 - \frac{ga}{2} = 2mga \left( 1 + \frac{1}{2} \right) \Rightarrow w = \left( \frac{7ga}{2} \right)^{\frac{1}{2}}$ <p><i>Alternatives using projectile motion from P</i></p> $v_P = \left( \frac{ga}{2} \right)^{\frac{1}{2}}, \text{ as above}$ $\downarrow u_y = \left( \frac{ga}{2} \right)^{\frac{1}{2}} \sin 60^\circ = \left( \frac{3ga}{8} \right)^{\frac{1}{2}}$ $\downarrow v_y^2 = u_y^2 + 2g \times \frac{3a}{2} = \frac{27ga}{8}$ $\rightarrow u_x = \left( \frac{ga}{2} \right)^{\frac{1}{2}} \cos 60^\circ = \left( \frac{ga}{8} \right)^{\frac{1}{2}}$ $w^2 = u_x^2 + v_y^2 = \frac{ga}{8} + \frac{27ga}{8} = \frac{7ga}{2} \Rightarrow w = \left( \frac{7ga}{2} \right)^{\frac{1}{2}}$ <p><i>There are also longer projectile methods using time of flight</i></p> <p>In outline, solving <math>\frac{3a}{2} = \left( \frac{3ga}{8} \right)^{\frac{1}{2}} t + \frac{1}{2}gt^2</math> gives <math>t = \left( \frac{3a}{2g} \right)^{\frac{1}{2}}</math>,</p> <p>then, using <math>v = u + at</math> gives <math>v_y = \left( \frac{3ga}{8} \right)^{\frac{1}{2}} + g \left( \frac{3a}{2g} \right)^{\frac{1}{2}} = \left( \frac{27ga}{8} \right)^{\frac{1}{2}}</math>, then as before.</p>	<p>M1 A1 <u>A1</u></p> <p>A1 (4)</p> <p>M1, A1</p> <p>A1</p> <p>A1 (4)</p> <p>M1 A1</p>

Question Number	Scheme	Marks
6.	<p>(a) <math>a = 3, T = 12</math> (or <math>\frac{1}{2}T = 6</math>)</p> $T = \frac{2\pi}{\omega} = 12 \Rightarrow \omega = \frac{\pi}{6} \quad (\square 0.52)$ <p>In the scheme below, when <math>a</math> and/or <math>\omega</math> appear in a line, accept the symbols or the candidates' values of <math>a</math> and/or <math>\omega</math> for the marks in that line.</p> <p>(Taking <math>x = a</math> when <math>t = 0</math>) <math>x = a \cos \omega t</math> M1</p> $\dot{x} = -a\omega \sin \omega t$ M1 A1 <p>When <math>t = 5</math> <math>\dot{x} = -3 \times \frac{\pi}{6} \sin \frac{5\pi}{6}</math> M1</p> $ \dot{x}  = \frac{\pi}{4} \quad (\text{m h}^{-1})$ awrt 0.79 A1 (9) <p>(b) Depth of 5.5 m <math>\Rightarrow x = -1.5</math></p> $-1.5 = a \cos \omega t$ M1 $\cos \omega t = -\frac{1}{2}$ A1ft $\frac{\pi}{6}t = \frac{2\pi}{3}, \left(\frac{4\pi}{3}\right)$ M1 $t = 4, 8$ A1 <p>Required time is <math>t_2 - t_1 = 8 - 4 = 4</math> (h) A1 (5)</p> <p>In 6(b), the following should be accepted</p> $1.5 = a \cos \omega t$ M1 $\cos \omega t = \frac{1}{2}$ A1ft $\frac{\pi}{6}t = \frac{\pi}{3}$ M1 $t = 2$ A1 <p>Required time is <math>2t = 4</math> (h) A1 (5)</p> <p><i>Further alternatives are given over the page.</i></p>	

Question Number	Scheme	Marks
6.	<p><i>Alternative to 6(a)</i> The last 5 marks of 6(a) can be gained as follows. The first 4 marks are as above.</p> <p>When <math>t=5</math></p> $x = 3 \cos \frac{5\pi}{6} = -\frac{3\sqrt{3}}{2} \quad (\approx -2.60)$ $v^2 = \omega^2 (a^2 - x^2)$ $= \frac{\pi^2}{6^2} \left( 9 - \frac{9 \times 3}{4} \right) \quad \left( = \frac{\pi^2}{16} \right)$ $ v  = \frac{\pi}{4} \quad (\text{m h}^{-1})$ <p style="text-align: right;">awrt 0.79</p> <p><i>Alternatives measuring <math>x</math> from the centre of oscillation</i></p> <p>(a) (Using 1400 as <math>t = 0</math>) The first 4 marks are as above</p> $x = a \sin \omega t$ $\dot{x} = a\omega \cos \omega t$ <p>When <math>t = 2</math></p> $\dot{x} = 3 \times \frac{\pi}{6} \cos \frac{2\pi}{6} \quad t=2 \text{ oe is essential for this M}$ $= \frac{\pi}{4} \quad (\text{m h}^{-1})$ <p>(b)</p> $1.5 = 3 \sin \omega t$ $\sin \omega t = \frac{1}{2}$ $\frac{\pi}{6} t = \frac{\pi}{6}, \quad \left( \frac{5\pi}{6} \right)$ $t = 1, 5$ <p>Required time is <math>t_2 - t_1 = 5 - 1 = 4 \quad (\text{h})</math></p>	<p>M1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p>B1 B1 M1 A1 M1 M1 A1</p> <p>M1</p> <p>A1 (9)</p> <p>M1</p> <p>A1ft</p> <p>M1</p> <p>A1</p> <p>A1 (5) [14]</p>

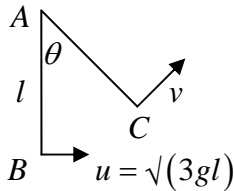
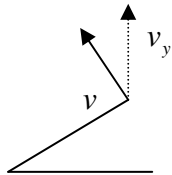
Question Number	Scheme	Marks
7.	<p>(a)</p> $\frac{1}{3}\ddot{x} = -\frac{k}{(x+1)^2}$ $\frac{1}{3}v \frac{dv}{dx} = -\frac{k}{(x+1)^2}$ $\int v dv = \int -\frac{3k}{(x+1)^2} dx \quad \text{Separating variables \&}$ $\frac{1}{2}v^2 = \frac{3k}{x+1} (+C) \quad \text{attempting integration of both sides}$ $v^2 = \frac{6k}{x+1} + A$ <p>Using boundary values to obtain two simultaneous equations.</p> $(1, 4) \quad 16 = 3k + A$ $(8, \sqrt{2}) \quad 2 = \frac{2k}{3} + A$ $14 = \frac{7}{3}k \Rightarrow k = 6$ <p>(b)</p> $A = -2$ $v^2 = \frac{36}{x+1} - 2 = 0$ $x = 17 \text{ (m)}$	<p>M1</p> <p>M1</p> <p>M1 A1=A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1 A1 (10)</p> <p>B1</p> <p>M1</p> <p>M1 A1 (4)</p> <p>[14]</p>

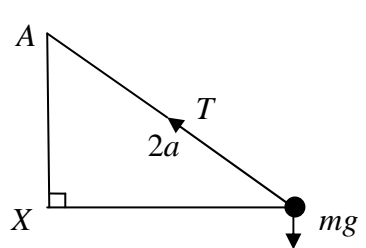
Question Number	Scheme	Marks
1.	<p>(a) <math>\rightarrow F = T \sin 60^\circ \quad \uparrow \quad T \cos 60^\circ = 0.8g</math> both  [or Z <math>F \cos 60^\circ = 0.8g \cos 30^\circ</math>]  <math>F = 0.8g \tan 60^\circ \approx 14 \text{ (N)}</math> accept 13.6</p> <p>(b) <math>T = \frac{0.8g}{\sin 30^\circ} (= 15.68)</math> allow in (a)  HL <math>15.68 = \frac{24 \times x}{1.2} \Rightarrow x \approx 0.78 \text{ (cm)}</math> accept 0.784</p> <p>(c) <math>E = \frac{24 \times x^2}{2 \times 1.2} \approx 6.1 \text{ (J)}</math> accept 6.15</p>	<p>M1 (M2) M1 A1 (3)</p> <p>M1 M1 A1 (3)</p> <p>M1 A1ft (2)</p> <p><b>Total 8 marks</b></p>
2.	<p>(a) <math>\frac{dv}{dt} = 2 \sin \frac{1}{2}t \Rightarrow v = A - 4 \cos \frac{1}{2}t</math>  <math>v = 4, t = 0 \Rightarrow 4 = A - 4 \Rightarrow A = 8</math>  <math>v = 8 - 4 \cos \frac{1}{2}t</math></p> <p>(b) <math>\int_{\dots}^{\dots} \left( 8 - 4 \cos \frac{1}{2}t \right) dt = 8t - 8 \sin \frac{1}{2}t</math> ft constants  <math>[...]_{\dots}^{\dots} = 4(\pi - \sqrt{2})</math> awrt 6.9</p>	<p>M1 A1 M1 A1 (4)</p> <p>M1 A1ft M1 A1 (4)</p> <p><b>Total 8 marks</b></p>

Question Number	Scheme	Marks
3.	<p>(a) N2L <math>ma = -\frac{cm}{x^2}</math></p> $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -\frac{c}{x^2} \Rightarrow \frac{1}{2}v^2 = A + \frac{c}{m}$ $v^2 = B + \frac{2c}{m}$ $x = R, v = U \Rightarrow B = U^2 - \frac{2c}{R}$ <p>Leading to <math>v^2 = U^2 + 2c\left(\frac{1}{x} - \frac{1}{R}\right)</math> *</p> <p>(b) <math>\frac{1}{2}\left[\frac{1}{2}mU^2\right] = \frac{1}{2}m\left[U^2 + 2c\left(\frac{1}{2R} - \frac{1}{R}\right)\right]</math></p> <p>Leading to <math>c = \frac{1}{2}RU^2</math></p>	<p>B1</p> <p>M1 A1 ignore A</p> <p>M1</p> <p>A1 cso</p> <p>(5)</p> <p>M1 A1</p> <p>A1</p> <p>(3)</p> <p><b>Total 8 marks</b></p>
4.	<p>(a) <math>5M\bar{x} = 3M \times \frac{h}{2} + 2M\left(h + \frac{3}{8}r\right)</math></p> $5\bar{x} = \frac{3h}{2} + 2h + \frac{3}{4}r = \frac{7h}{2} + \frac{3}{4}r$ $\bar{x} = \frac{14h + 3r}{20} *$ <p>(b) </p> $\tan \alpha = \frac{20r}{14h + 3r} = \frac{4}{3}$ <p>Leading to <math>h = \frac{6}{7}r</math></p>	<p>M1 A2(1,0)</p> <p>M1 A1</p> <p>(5)</p> <p>M1 A1</p> <p>M1 A1</p> <p>(4)</p> <p><b>Total 9 marks</b></p>



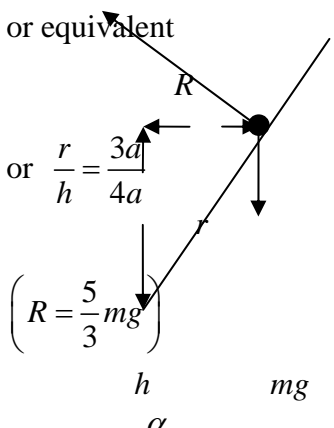
Question Number	Scheme	Marks
5.	<div style="text-align: center;"> </div> <p>(a) HL <math>T = mg = \frac{\lambda \times \frac{1}{4}l}{l} \Rightarrow \lambda = 4mg</math></p> <p>(b) N2L <math>mg - T = m\ddot{x}</math>  <math>mg - \frac{4mg(\frac{1}{4}l + x)}{l} = m\ddot{x}</math>  <math>\frac{d^2x}{dt^2} = -\frac{4g}{l}x</math> *</p> <p>(c) <math>v^2 = \omega^2(a^2 - x^2) = \frac{4g}{l}\left(\frac{l^2}{4} - \frac{l^2}{16}\right)</math>          Leading to <math>v = \frac{1}{2}\sqrt{3gl}</math></p> <p>or energy, <math>\frac{1}{2} \frac{4mg \cdot gl^2/16}{l} = \frac{1}{2}mv^2 + mg \cdot \frac{3l}{4}</math> for the first M1 A1 in (c)</p> <p>(d) <math>P</math> first moves freely under gravity,          then (part) SHM.</p>	<p>M1 A1 (2)</p> <p>M1 M1 A1 M1 A1 (5)</p> <p>M1 A1 M1 A1 (4)</p> <p>B1 B1 (2)</p> <p><b>Total 13 marks</b></p>

Question Number	Scheme	Marks
6.	<p>(a)</p>  <p>Energy <math>\frac{1}{2}m(u^2 - v^2) = mgl(1 - \cos \theta)</math>  <math>[v^2 = gl + 2gl \cos \theta]</math></p> <p>N2L <math>T - mg \cos \theta = \frac{mv^2}{l}</math>  <math>= \frac{mg\lambda(1 + 2\cos \theta)}{\lambda}</math>  <math>T = mg(1 + 3\cos \theta) *</math></p> <p>(b)</p> $T = 0 \Rightarrow \cos \theta = -\frac{1}{3}$ $v^2 = gl - \frac{2}{3}gl \Rightarrow v = \left(\frac{gl}{3}\right)^{\frac{1}{2}}$ <p>(c)</p>  $\uparrow v_y = \left(\frac{gl}{3}\right)^{\frac{1}{2}} \sin \theta \left[ = \left(\frac{gl}{3}\right)^{\frac{1}{2}} \cdot \frac{2\sqrt{2}}{3} \right]$ $v^2 = u^2 - 2gh \Rightarrow 2gh = \frac{gl}{3} \cdot \frac{8}{9} \Rightarrow h = \frac{4l}{27}$ $H = l(1 - \cos \theta) + \frac{4l}{27} = \frac{40l}{27}$	<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>cs0 A1</p> <p>(6)</p> <p>B1</p> <p>M1 A1</p> <p>(3)</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>(5)</p> <p><b>Total 14 marks</b></p>

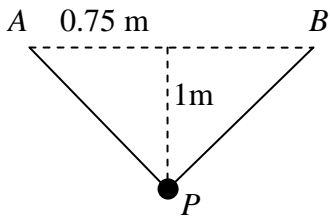
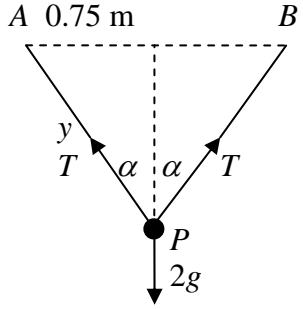
Question Number	Scheme	Marks
7.	<p>(a) N2L <math>\leftarrow T \cos 30^\circ = m(2a \cos 30^\circ) \left( \frac{kg}{3a} \right)</math></p> $T = \frac{2kmg}{3} *$ <p style="text-align: right;">cso</p> <p>(b) <math>\uparrow R = mg - T \sin 30^\circ</math></p> $= mg \left( 1 - \frac{k}{3} \right)$ <p>(c) <math>(R \geq 0) \Rightarrow k \leq 3</math> ignore <math>k &gt; 0</math>, accept <math>k &lt; 3</math></p> <p>(d)</p>  <p>N2L <math>\leftarrow T \cos \theta = m(2a \cos \theta) \left( \frac{2g}{a} \right)</math></p> $(T = 4mg)$ <p><math>\uparrow T \sin \theta = mg</math></p> <p>Eliminating T</p> $AX = 2a \sin \theta = \frac{1}{2}a$ <p><math>AO = 2a \sin 30^\circ = a \Rightarrow AX = \frac{1}{2}AO</math>, as required *</p> <p style="text-align: right;">cso</p>	<p>M1 A1</p> <p>A1</p> <p>(3)</p> <p>M1 A1</p> <p>A1</p> <p>(3)</p> <p>M1 A1</p> <p>(2)</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1, A1</p> <p>(7)</p> <p><b>Total 15 marks</b></p>

June 2006  
6679 Mechanics M3  
Mark Scheme

Question Number	Scheme	Marks
1.	<p>Use of <math>(\pi) \int y^2 dx \times \bar{x} = (\pi) \int xy^2 dx</math></p> $\int x dx \times \bar{x} = \int x^2 dx$ $\left[ \frac{1}{2} x^2 \right]_{\dots}^{\dots} \times \bar{x} = \left[ \frac{1}{3} x^3 \right]_{\dots}^{\dots}$ <p>Using limits 0 and 4</p> $\frac{16}{2} \times \bar{x} = \frac{64}{3}$ $\bar{x} = \frac{8}{3}$	<p>M1</p> <p>A1 = A1</p> <p>M1</p> <p>A1 (5)</p> <p>[5]</p>
2.	<p>(a)                      Small Hemisphere      Bowl      Large Hemisphere</p> <p>Mass ratios              <math>\frac{2}{3} \pi \left( \frac{a}{2} \right)^3</math>      <math>\frac{2}{3} \pi \frac{7a^3}{8}</math>      <math>\frac{2}{3} \pi a^3</math></p> <p style="text-align: right;">Anything in the ratio 1 : 7 : 8</p> <p><math>\bar{x}</math>                      <math>\frac{3}{16} a</math>                      <math>\bar{x}</math>                      <math>\frac{3}{8} a</math></p> $1 \times \frac{3}{16} a + 7 \times \bar{x} = 8 \times \frac{3}{8} a$ <p>Leading to              <math>\bar{x} = \frac{45}{112} a</math> *                      cso</p> <p>(b)                      Bowl      Liquid      Bowl and Liquid</p> <p>Mass Ratios              <math>M</math>              <math>kM</math>              <math>(k+1)M</math></p> <p><math>\bar{x}</math>                      <math>\frac{45}{112} a</math>              <math>\frac{3}{16} a</math>              <math>\frac{17}{48} a</math></p> $M \times \frac{45}{112} a + kM \times \frac{3}{16} a = (k+1)M \times \frac{17}{48} a$ <p>Leading to              <math>k = \frac{2}{7}</math></p>	<p>B1</p> <p>B1</p> <p>M1 A1</p> <p>A1 (5)</p> <p>B1</p> <p>B1</p> <p>M1 A1</p> <p>A1 (5)</p> <p>[10]</p>

Question Number	Scheme	Marks
3.	<p>(a)</p> $a = 0.1$ $\frac{2\pi}{\omega} = \frac{1}{5} \Rightarrow \omega = 10\pi$ $F_{\max} = ma\omega^2$ $= 0.2 \times 0.1 \times (10\pi)^2$ $\approx 19.7 \text{ (N)}$ <p>cao</p> <p>(b)</p> $a' = 0.2, \quad \omega' = 10\pi$ $v^2 = \omega^2 (a^2 - x^2) = 100\pi^2 (0.2^2 - 0.1^2) \quad (= 3\pi^2 \approx 29.6 \dots)$ $v \approx 5.44 \text{ (ms}^{-1}\text{)}$ <p>cao</p> <p><i>If answers are given to more than 3 significant figures a maximum of one A mark is lost in the question.</i></p>	<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p><b>(6)</b></p> <p>B1ft, B1ft</p> <p>M1 A1</p> <p>A1</p> <p><b>(5)</b></p> <p><b>[11]</b></p>
4.	<p>or equivalent</p>  <p>or <math>\frac{r}{h} = \frac{3a}{4a}</math></p> <p><math>\left(R = \frac{5}{3}mg\right)</math></p> <p><math>h</math> <math>mg</math> <math>\alpha</math></p> <p><math>= mr \times \frac{8g}{9a} \left(R = \frac{10mrg}{9a}\right)</math></p> <p><math>\tan \alpha = \frac{9a}{8r} \left(\frac{5}{3}mg = \frac{10mrg}{9a}\right)</math></p> <p>Eliminating <math>R</math></p> <p><math>\left(\frac{3}{4} = \frac{9a}{8r} \Rightarrow r = \frac{3}{2}a\right)</math></p> <p><math>h = \frac{r}{\tan \alpha} = \frac{3a}{2} \times \frac{4}{3} = 2a</math></p>	<p><math>\tan \alpha = \frac{3}{4}</math></p> <p>B1</p> <p><math>\tan \alpha = \frac{r}{h}</math></p> <p>B1</p> <p><math>R(\uparrow) \quad R \sin \alpha = mg</math></p> <p>M1 A1</p> <p><math>R(\leftarrow) \quad R \cos \alpha = mr\omega^2</math></p> <p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1</p> <p><b>(11)</b></p>

		<b>[11]</b>
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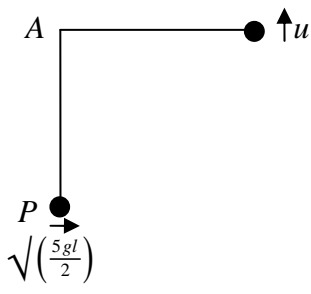
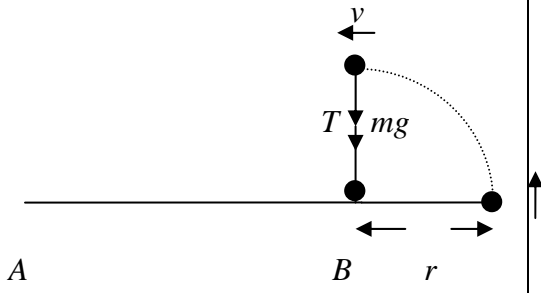
Question Number	Scheme	Marks
5.	<p>(a)</p>  $AP = \sqrt{(0.75^2 + 1^2)} = 1.25$ <p>Conservation of energy</p> $\frac{1}{2} \times 2 \times v^2 + 2 \times \frac{49 \times 0.5^2}{2 \times 0.75} = 2g \times 1 \quad -1$ <p>for each incorrect term</p> <p>Leading to <math>v \approx 1.8 \text{ (ms}^{-1}\text{)}</math></p> <p>accept 1.81</p> <p>(b)</p>  <p> <math>R(\uparrow) \quad 2T \cos \alpha = 2g</math>  <math>y = \frac{0.75}{\sin \alpha}</math> </p> <p>Hooke's Law</p> $T = \frac{49}{0.75} \left( \frac{0.75}{\sin \alpha} - 0.75 \right)$ $= 49 \left( \frac{1}{\sin \alpha} - 1 \right)$ $\frac{9.8}{\cos \alpha} = 49 \left( \frac{1}{\sin \alpha} - 1 \right)$ <p>Eliminating <math>T</math></p> $\tan \alpha = 5(1 - \sin \alpha)$ $5 = \tan \alpha + 5 \sin \alpha \quad *$ <p>cs0</p>	<p>M1 A1</p> <p>M1 A2 (1, 0)</p> <p>A1 (6)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (6)</p> <p>[12]</p>





Question Number	Scheme	Marks
6.	<p>(a)</p> <p>Parabola</p> <p>Hyperbola</p> <p>Points</p> <p>(b) Identifying the minimum point of the parabola and 5 as the end points.</p> $2 < t < 5$ <p>(c) Splitting the integral into two part, with limits 0 and 4, and 4 and 5, and evaluating both integrals.</p> $\int_0^4 3t(t-4)dt = [t^3 - 6t^2]_0^4 = -32 \quad \text{and} \quad \int_4^5 3t(t-4)dt = [t^3 - 6t^2]_4^5 = 7$ <p>Both</p> <p>Total distance = 39 (m) *</p> <p>cs0</p> <p>(d)</p> $\int_5^{t_1} \frac{75}{t} dt = 32 - 7$ $75[\ln t]_5^{t_1} = 25$ $\ln \frac{t_1}{5} = \frac{1}{3} \Rightarrow t_1 = 5e^{\frac{1}{3}}$ $\approx 6.98$ <p>cao</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p> <p>M1</p> <p>A1</p> <p>(2)</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>M1 A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>(5)</p> <p>[13]</p>

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Question Number	Scheme	Marks
7.	<p>(a)</p>  <p>Conservation of Energy</p> $\frac{1}{2}m\left(\frac{5gl}{2} - u^2\right) = mgl$ <p>Leading to <math>u = \sqrt{\left(\frac{gl}{2}\right)}</math></p> <p>(b)</p>  <p>Conservation of Energy</p> $\frac{1}{2}m(u^2 - v^2) = mgr$ $v^2 = u^2 - 2gr$ $R(\downarrow) \quad T + mg = \frac{mv^2}{r}$ $T = \frac{m}{r}(u^2 - 2gr) - mg$ $= \frac{mu^2}{r} - 3mg$ $= \frac{mgl}{2r} - 3mg$ $T \geq 0 \Rightarrow \frac{mgl}{2r} \geq 3mg$ $\Rightarrow \frac{1}{6} \geq r$ $AB_{\text{MIN}} = \frac{5l}{6}$	<p>M1 A1= A1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1 (9)</p> <p>[13]</p>







# Mark Scheme (Results)

## January 2007

GCE

GCE Mathematics

Mechanics M3 (6679)

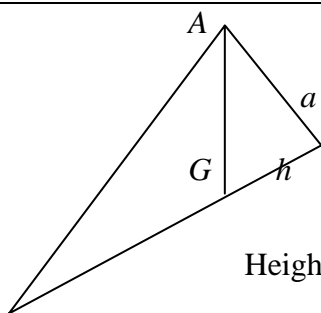


January 2007  
6679 Mechanics M3  
Mark Scheme

Question Number	Scheme	Marks
1.	<p>(a) Maximum speed when accel. = 0 (o.e.)</p> <p>(b) <math>\frac{1}{12}(30 - x) = v \frac{dv}{dx}</math> (acceln = ... + attempt to integrate)</p> <p>Use of <math>v \frac{dv}{dx}</math>: <math>\frac{v^2}{2} = \frac{1}{12} \left( 30x - \frac{x^2}{2} \right) (+ c)</math></p> <p>Substituting <math>x = 30</math>, <math>v = 10</math> and finding <math>c</math> (= 12.5), or limits</p> <p><u><math>v^2 = 25 + 5x - \frac{1}{12}x^2</math> (o.e.)</u></p> <p>(a) Allow “acceln &gt; 0 for <math>x &lt; 30</math>, acceln &lt; 0 for <math>x &gt; 30</math>” Also “accelerating for <math>x &lt; 30</math>, decelerating for <math>x &gt; 30</math>” But “acceln &lt; 0 for <math>x &gt; 30</math>” only is B0</p> <p>(b) 1<sup>st</sup> M1 will be generous for wrong form of acceln (e.g. <math>dv/dx</math>)! 3<sup>rd</sup> M1 If use limits, they must use them in correct way with correct values Final A1. Have to accept any expression, but it must be for <math>v^2</math> explicitly (not <math>1/2v^2</math>), and if in separate terms, one can expect like terms to be collected. Hence answer in form as above, or e.g. <math>\frac{1}{12}(300 + 60x - x^2)</math>; also <math>100 - \frac{1}{12}(30 - x)^2</math></p>	<p>B1 (1)</p> <p>M1 ↓ M1 A1  ↓ M1</p> <p>A1 (5)</p>



2.



$$\text{Height of cone} = \frac{a}{\tan \alpha} = 3a$$

$$\text{Hence } h = \frac{3}{4}a$$

$$\tan \theta = \frac{a}{\frac{3}{4}a} = \frac{4}{3} \Rightarrow \theta = 53.1^\circ$$

1<sup>st</sup> M1 (generous) allow any trig ratio to get height of cone (e.g. using sin)

3<sup>rd</sup> M1 For correct trig ratio on a suitable triangle to get  $\theta$  or complement (even if they call the angle by another name – hence if they are aware or not that they are getting the required angle)

M1 A1

↓

M1

↓

M1 A1

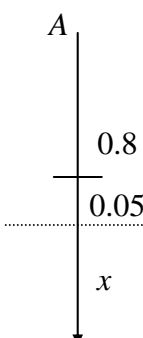
(5)

3.	<p>(a) <math display="block">\text{E.P.E.} = \frac{1}{2} \frac{3.6mg}{a} x^2 = \frac{1}{2} \frac{3.6mg}{a} \left(\frac{a}{3}\right)^2</math>  <math display="block">= \underline{0.2 mga}</math></p> <p>(b) Friction = <math>\mu mg \Rightarrow</math> work done by friction = <math>\mu mg \left(\frac{4a}{3}\right)</math></p> <p>Work-energy: <math>\frac{1}{2} m.2ga = \mu mgd + 0.2 mga</math> (3 relevant terms)</p> <p>Solving to find <math>\mu</math>: <u><math>\mu = 0.6</math></u></p> <p>(b) 1<sup>st</sup> M1: allow for attempt to find work done by frictional force (i.e. not just finding friction).  2<sup>nd</sup> M1: “relevant” terms, i.e. energy or work terms!  A1 f.t. on their work done by friction</p>	<p>M1 A1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>M1 A1✓ ↓</p> <p>M1 A1 (6)</p>
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4.	<p>(a) Energy: <math>\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mga(1 + \cos \theta)</math></p> $\underline{v^2 = ag(1 - 2\cos \theta)} \quad (\text{o.e.})$ <p>(b) <math>T + mg \cos \theta = m \frac{v^2}{a}</math></p> $\text{Hence } \underline{T = (1 - 3\cos \theta)mg} \quad (*)$ <p>(c) Using <math>T = 0</math> to find <math>\cos \theta</math></p> <p>Hence height above <math>A = \underline{\frac{4}{3}a}</math> Accept <math>1.33a</math> (but must have 3+ s.f.)</p> <p>(d) <math>v^2 = \frac{1}{3}ag \quad (\text{o.e.})</math> f.t. using <math>\cos \theta = \frac{1}{3}</math> in <math>v^2</math></p> <p>consider vert motion: <math>(v \sin \theta)^2 = 2gh</math> (with <math>v</math> resolved)</p> <p><math>\sin^2 \theta = \frac{8}{9}</math> (or <math>\theta = 70.53</math>, <math>\sin \theta = 0.943</math>) and solve for <math>h</math> (as <math>ka</math>)</p> $h = \underline{\frac{4}{27}a} \text{ or } 0.148a \text{ (awrt)}$ <p><b>OR</b> consider energy: <math>\frac{1}{2}m(v \cos \theta)^2 + mgh = \frac{1}{2}mv^2</math> (3 non-zero terms)</p> <p>Sub for <math>v</math>, <math>\theta</math> and solve for <math>h</math></p> $h = \underline{\frac{4}{27}a} \text{ or } 0.148a \text{ (awrt)}$	<p>M1 A1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>A1 cso (3)</p> <p>M1</p> <p>A1 (2)</p> <p>B1√</p> <p>M1 A1 ↓ M1</p> <p>A1</p> <p>M1 A1 ↓ M1</p> <p>A1</p>
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Question Number	Scheme	Marks
5.	<p>(a) <math>\uparrow T \cos \theta = mg</math></p> <p><math>\leftrightarrow T + T \sin \theta = mr\omega^2</math> (3 terms)</p> <p><math>r = h \tan \theta</math></p> <p><math>\frac{mg}{\cos \theta}(1 + \sin \theta) = \frac{m\omega^2 h \sin \theta}{\cos \theta}</math> (eliminate <math>r</math>)</p> <p><math>\omega^2 = \frac{g}{h} \left( \frac{1 + \sin \theta}{\sin \theta} \right)</math> (*) (solve for <math>\omega^2</math>)</p> <p>(b) <math>\omega^2 = \frac{g}{h} \left( \frac{1}{\sin \theta} + 1 \right) &gt; \frac{2g}{h} (\sin \theta &lt; 1) \Rightarrow \omega &gt; \sqrt{\frac{2g}{h}}</math> (*)</p> <p>(c) <math>\frac{3g}{h} = \frac{g}{h} \left( \frac{1 + \sin \theta}{\sin \theta} \right) \Rightarrow \sin \theta = \frac{1}{2}</math></p> <p><math>T \cos \theta = mg \Rightarrow T = \frac{2\sqrt{3}}{3}mg</math> or <u>1.15mg</u> (awrt)</p> <p>(a) Allow first B1 M1 A1 if assume different tensions (so next M1 is effectively for eliminating <math>r</math> <b>and</b> <math>T</math>.)</p> <p>(b) M1 requires a <i>valid</i> attempt to derive an <i>inequality</i> for <math>\omega</math>. (Hence putting <math>\sin \theta = 1</math> immediately into expression of <math>\omega^2</math> [assuming this is the critical value] is M0.)</p>	<p>B1</p> <p>M1 A1</p> <p>B1</p> <p>↓ M1</p> <p>↓ M1 A1 (7)</p> <p>M1 A1 (2)</p> <p>M1 A1</p> <p>↓ M1 A1 (4)</p>

6.	<p>(a) Moments: <math>\pi \int_1^2 xy^2 dx = V \bar{x}</math> or <math>\int_1^2 xy^2 dx = \bar{x} \int_1^2 y^2 dx</math></p> $\int_1^2 y^2 dx = \int_1^2 \frac{1}{4x^4} dx = \left[ -\frac{1}{12x^3} \right]_1^2 \quad (= \frac{7}{96}) \quad (\text{either})$ $\int_1^2 xy^2 dx = \int_1^2 \frac{1}{4x^3} dx = \left[ -\frac{1}{8x^2} \right]_1^2 \quad (= \frac{3}{32}) \quad (\text{both})$ <p>Solving to find <math>\bar{x}</math> (<math>= \frac{9}{7}</math>) <math>\Rightarrow</math> required dist <math>= \frac{9}{7} - 1 = \frac{2}{7}</math> m (*)</p> <p>(b)</p> <table><tr><td></td><td><math>H</math></td><td><math>S</math></td><td><math>T</math></td></tr><tr><td>Mass</td><td><math>(\rho) \frac{2}{3} \pi \left( \frac{1}{2} \right)^3</math></td><td><math>(\rho) \frac{7\pi}{96}</math></td><td><math>H + S</math></td></tr><tr><td></td><td><math>\left[ = \frac{1}{12} (\rho) \pi \right]</math></td><td></td><td><math>\left[ = \frac{5}{32} (\rho) \pi \right]</math></td></tr></table> <p>Dist of CM from base <math>\frac{19}{16}</math> m <math>\frac{5}{7}</math> m <math>\bar{x}</math></p> <p>Moments: <math>\left[ = \frac{1}{12} (\rho) \pi \right] \left( \frac{19}{16} \right) + (\rho) \frac{7\pi}{96} \left( \frac{5}{7} \right) = \left[ \frac{5}{32} (\rho) \pi \right] \bar{x}</math></p> $\bar{x} = \frac{29}{30} \text{ m or } 0.967 \text{ m (awrt)}$ <p>Allow distances to be found from different base line if necessary</p>		$H$	$S$	$T$	Mass	$(\rho) \frac{2}{3} \pi \left( \frac{1}{2} \right)^3$	$(\rho) \frac{7\pi}{96}$	$H + S$		$\left[ = \frac{1}{12} (\rho) \pi \right]$		$\left[ = \frac{5}{32} (\rho) \pi \right]$	<p>M1</p> <p>M1 A1</p> <p>A1</p> <p>↓</p> <p>M1 A1 cso (6)</p> <p>B1, M1</p> <p>B1 B1</p> <p>M1 A1</p> <p>A1 (7)</p>
	$H$	$S$	$T$											
Mass	$(\rho) \frac{2}{3} \pi \left( \frac{1}{2} \right)^3$	$(\rho) \frac{7\pi}{96}$	$H + S$											
	$\left[ = \frac{1}{12} (\rho) \pi \right]$		$\left[ = \frac{5}{32} (\rho) \pi \right]$											

7.	<p>(a) </p> $T = \frac{\lambda}{0.8}(0.05) = 0.25g$ $\lambda = \frac{(0.8)(0.25g)}{0.05} = 39.2 \text{ (*)}$ <p>(b)</p> $T = \frac{39.2}{0.8}(x + 0.05)$ $mg - T = ma \quad (3 \text{ term equn})$ $0.25g - \frac{39.2}{0.8}(x + 0.05) = 0.25 \ddot{x} \text{ (or equivalent)}$ $\ddot{x} = -196x$ $\text{SHM with period } \frac{2\pi}{\omega} = \frac{2\pi}{14} = \frac{\pi}{7} \text{ s (*)}$ <p>(c)</p> $v = 14 \sqrt{\{(0.1)^2 - (0.05)^2\}}$ $= 1.21(24...) \approx \underline{1.21 \text{ m s}^{-1}} \text{ (3 s.f.) Accept } 7\sqrt{3}/10$ <p>(d) Time <math>T</math> under gravity = <math>\frac{1.21..}{g}</math> (= 0.1237 s)</p> <p>Complete method for time <math>T'</math> from <math>B</math> to slack.</p> <p>[ <math>\uparrow</math> e.g. <math>\frac{\pi}{28} + t</math>, where <math>0.05 = 0.1 \sin 14t</math></p> <p>OR <math>T'</math>, where <math>-0.05 = 0.1 \cos 14T'</math> ]</p> $T'' = 0.1496\text{s}$ $\text{Total time} = T + T' = \underline{0.273 \text{ s}}$ <p>(b) 1<sup>st</sup> M1 must have extn as <math>x + k</math> with <math>k \neq 0</math> (but allow M1 if e.g. <math>x + 0.15</math>), or must justify later</p> <p>For last four marks, <i>must</i> be using <math>\ddot{x}</math> (not <math>a</math>)</p> <p>(c) Using <math>x = 0</math> is M0</p> <p>(d) M1 – must be using distance for when string goes slack. Using <math>x = -0.1</math> (i.e. assumed end of the oscillation) is M0</p>	<p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1 ↓ M1 A1 cso (6)</p> <p>M1 A1√</p> <p>A1 (3)</p> <p>B1√</p> <p>M1 A1</p> <p>A1</p> <p>A1 (5)</p>
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# Mark Scheme (Results)

## Summer 2007

GCE

GCE Mathematics

Mechanics M3 (6679)

June 2007  
6679 Mechanics M3  
Mark Scheme

Question Number	Scheme	Marks
1.	<p>(a)</p> $A = \int_0^2 (2x - x^2) dx$ $= \left[ x^2 - \frac{x^3}{3} \right]_0^2$ $A = \left[ x^2 - \frac{x^3}{3} \right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3} \quad *$ <p style="text-align: right;">cs0</p> <p>(b)</p> $\bar{x} = 1 \quad (\text{by symmetry})$ $\frac{4}{3} \bar{y} = \frac{1}{2} \int y^2 dx = \frac{1}{2} \int (2x - x^2)^2 dx$ $= \frac{1}{2} \int (4x^2 - 4x^3 + x^4) dx$ $= \frac{1}{2} \left[ \frac{4x^3}{3} - x^4 + \frac{x^5}{5} \right]_0^2$ $\frac{4}{3} \bar{y} = \frac{1}{2} \left[ \frac{4x^3}{3} - x^4 + \frac{x^5}{5} \right]_0^2 = \frac{8}{15}$ $\bar{y} = \frac{8}{15} \times \frac{3}{4} = \frac{2}{5}$ <p style="text-align: right;">accept exact equivalents</p>	<p>M1 A1</p> <p>A1</p> <p>A1 (4)</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1 (5)</p> <p>[9]</p>



Question Number	Scheme					Marks
2.	(a)	Base	Cylinder	Container		
	Mass ratios	$\pi h^2$	$2\pi h^2$	$3\pi h^2$	Ratio of 1 : 2 : 3	B1
	$\bar{y}$	0	$\frac{h}{2}$	$\bar{y}$		B1

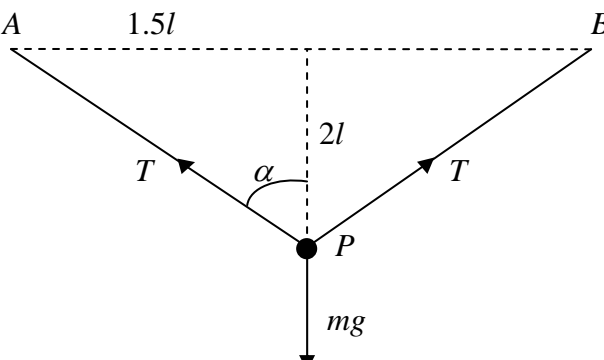
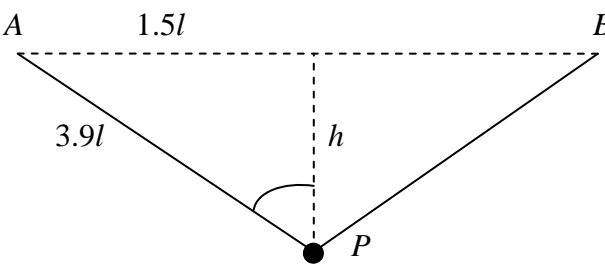
Question Number	Scheme	Marks
3.	<p>(a) At surface</p> $\frac{k}{R^2} = mg \Rightarrow k = mgR^2 \quad *$ <p>(b) N2L</p> $m\ddot{x} = -\frac{mgR^2}{x^2}$ $v \frac{dv}{dx} = -\frac{gR^2}{x^2} \quad \text{or} \quad \frac{d}{dx} \left( \frac{1}{2} v^2 \right) = -\frac{gR^2}{x^2}$ $\int v dv = -gR^2 \int \frac{1}{x^2} dx \quad \text{or} \quad \frac{1}{2} v^2 = -gR^2 \int \frac{1}{x^2} dx$ $\frac{1}{2} v^2 = \frac{gR^2}{x} (+C)$ $x = 2R, v = 0 \Rightarrow C = -\frac{gR}{2}$ $v^2 = \frac{2gR^2}{x} - gR$ <p>At <math>x = R</math>,</p> $v^2 = \frac{2gR^2}{R} - gR$ $v = \sqrt{(gR)}$	<p>cs0 M1 A1 (2)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p> <p>[9]</p>

Question Number	Scheme	Marks
4.	<div data-bbox="606 347 965 862" data-label="Image"> </div> <div data-bbox="510 896 845 952" data-label="Equation-Block"> <math display="block">\uparrow \quad T \cos \theta = mg</math> </div> <div data-bbox="510 952 845 1041" data-label="Equation-Block"> <math display="block">\leftarrow \quad T \sin \theta = \frac{mv^2}{r}</math> </div> <div data-bbox="606 1041 845 1131" data-label="Equation-Block"> <math display="block">\tan \theta = \frac{r}{\sqrt{l^2 - r^2}}</math> </div> <div data-bbox="1053 1052 1244 1097" data-label="Text"> <p>or equivalent</p> </div> <div data-bbox="598 1164 750 1265" data-label="Equation-Block"> <math display="block">\tan \theta = \frac{v^2}{rg}</math> </div> <div data-bbox="1029 1187 1236 1243" data-label="Text"> <p>Eliminating <math>T</math></p> </div> <div data-bbox="534 1265 750 1366" data-label="Equation-Block"> <math display="block">\frac{r}{\sqrt{l^2 - r^2}} = \frac{v^2}{rg}</math> </div> <div data-bbox="1029 1276 1236 1332" data-label="Text"> <p>Eliminating <math>\theta</math></p> </div> <div data-bbox="622 1366 925 1422" data-label="Equation-Block"> <math display="block">gr^2 = v^2 \sqrt{l^2 - r^2} \quad *</math> </div> <div data-bbox="1173 1377 1236 1411" data-label="Text"> <p>cs0</p> </div>	<div data-bbox="1276 896 1380 952" data-label="Text"> <p>M1 A1</p> </div> <div data-bbox="1276 963 1380 1019" data-label="Text"> <p>M1 A1</p> </div> <div data-bbox="1276 1052 1380 1108" data-label="Text"> <p>M1 A1</p> </div> <div data-bbox="1276 1187 1332 1243" data-label="Text"> <p>M1</p> </div> <div data-bbox="1276 1288 1332 1344" data-label="Text"> <p>M1</p> </div> <div data-bbox="1276 1366 1460 1467" data-label="Text"> <p>A1 <b>(9)</b> <b>[9]</b></p> </div>

Question Number	Scheme	Marks
5.	<p>(a) <math>\ddot{x} = -\omega^2 x \Rightarrow 1 = \omega^2 \times 0.04 \quad (\Rightarrow \omega = 5)</math></p> $T = \frac{2\pi}{5}$ <p style="text-align: right;">awrt 1.3</p>	<p>M1 A1</p> <p>A1 (3)</p>
	<p>(b) <math>v^2 = \omega^2 (a^2 - x^2) \Rightarrow 0.2^2 = 5^2 (a^2 - 0.04^2)</math></p> $a = \frac{\sqrt{2}}{25}$ <p style="text-align: right;">ft their <math>\omega</math></p> <p style="text-align: right;">accept exact equivalents or awrt 0.057</p>	<p>M1 A1ft</p> <p>A1 (3)</p>
	<p>(c) Using <math>x = a \cos \omega t</math></p> $\frac{1}{2}a = a \cos \omega t$ $5t = \frac{\pi}{3}$ $t = \frac{\pi}{15}$ $T' = 4t = \frac{4\pi}{15}$ <p style="text-align: right;">ft their <math>\omega</math></p> <p style="text-align: right;">awrt 0.84</p>	<p>M1 A1ft</p> <p>A1</p> <p>M1 A1 (5)</p> <p>[11]</p>
	<p><i>Alternative to (c)</i></p> <p>Using <math>x = a \sin \omega t</math></p> $\frac{1}{2}a = a \sin \omega t$ $5t = \frac{\pi}{6}$ $t = \frac{\pi}{30}$ $T' = T - 4t = \frac{4\pi}{15}$ <p style="text-align: right;">ft their <math>\omega</math></p> <p style="text-align: right;">awrt 0.84</p>	<p>M1 A1ft</p> <p>A1</p> <p>M1 A1 (5)</p>

Question Number	Scheme	Marks
6.	<div data-bbox="571 392 965 795" data-label="Image"> </div> <p>(a) Energy <math>\frac{1}{2}m(U^2 - v^2) = mga(1 + \cos \alpha)</math></p> <p><math>\square \quad (T +) \quad mg \cos \alpha = \frac{mv^2}{a}</math></p> <p>Leaves circle when <math>T = 0</math></p> <p><math>g \cos \alpha = \frac{U^2 - 2ga - 2ga \cos \alpha}{a}</math>      Eliminating <math>v</math></p> <p>Leading to <math>U^2 = ag(2 + 3 \cos \alpha) \quad *</math>      cso</p> <p>(b) Using conservation of energy from the lowest point of the surface</p> <p><math>\frac{1}{2}m(U^2 - W^2) = mga</math></p> <p><math>W^2 = U^2 - 2ag</math></p> <p>Using <math>\cos \alpha = \frac{1}{\sqrt{3}}, \quad W^2 = ag\left(2 + \frac{3}{\sqrt{3}}\right) - 2ag</math></p> <p><math>= ag\sqrt{3} \quad *</math>      cso</p> <p><i>Alternatives for (b) are given on the next page.</i></p>	<p>M1 A1=A1</p> <p>M1 A1</p> <p>M1</p> <p>A1      (7)</p> <p>M1 A1=A1</p> <p>M1</p> <p>A1      (5)</p> <p>[12]</p>

Question Number	Scheme	Marks
6.	<p><i>Alternative to part (b) using conservation of energy from the point where P loses contact with surface.</i></p> $\left( V^2 = ag \cos \alpha = \frac{ga}{\sqrt{3}} \right)$ <p>Energy <math>\frac{1}{2}m(W^2 - V^2) = mga \cos \alpha</math></p> $\frac{1}{2}m\left(W^2 - \frac{1}{\sqrt{3}}ag\right) = mga \times \frac{1}{\sqrt{3}}$ <p>Leading to <math>W^2 = ag \sqrt{3} \quad *</math></p> <p><i>Alternative to part (b) using projectile motion from the point where P loses contact with surface.</i></p> $V^2 = ag \cos \alpha = \frac{ga}{\sqrt{3}}$ <p>↓ <math>W_y^2 = V^2 \sin^2 \alpha + 2ga \cos \alpha</math></p> $= \frac{1}{\sqrt{3}}ag \left(1 - \frac{1}{3}\right) + 2ga \times \frac{1}{\sqrt{3}} = \frac{8\sqrt{3}}{9}ag$ <p>← <math>V_x = V \cos \alpha</math></p> $W^2 = W_y^2 + V_x^2 = \frac{8\sqrt{3}}{9}ag + \frac{1}{3}ag \sqrt{3} \times \frac{1}{3} = ag \sqrt{3} \quad *$	<p>M1 A1</p> <p>A1</p> <p>M1 A1 (5)</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1 (5)</p>

Question Number	Scheme	Marks
7.	<p>(a)</p>  $AP = \sqrt{(1.5l)^2 + (2l)^2} = 2.5l$ $\cos \alpha = \frac{4}{5}$ <p>Hooke's Law <math>T = \frac{\lambda(2.5l - 1.5l)}{1.5l} \left( = \frac{2\lambda}{3} \right)</math></p> <p style="margin-left: 100px;">↑ <math>2T \cos \alpha = mg \quad \left( T = \frac{5mg}{8} \right)</math></p> $2 \times \frac{2\lambda}{3} \times \frac{4}{5} = mg \quad \left( \frac{2\lambda}{3} = \frac{5mg}{8} \right)$ $\lambda = \frac{15mg}{16} *$ <p>(b)</p>  $h = \sqrt{(3.9l)^2 - (1.5l)^2} = 3.6l$ <p>Energy <math>\frac{1}{2}mv^2 + mg \times h = 2 \times \frac{15mg}{16} \times \frac{(2.4l)^2}{2 \times 1.5l}</math></p> <p>Leading to <math>v = 0 *</math></p>	<p>M1 A1</p> <p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>cs0 A1 (9)</p> <p>M1 A1</p> <p>M1 A1 ft = A1</p> <p>A1 (6) [15]</p>

# Mark Scheme (Results)

## January 2008

GCE

GCE Mathematics (6679/01)

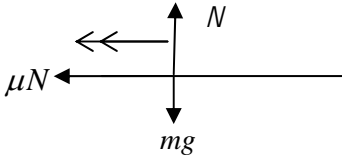
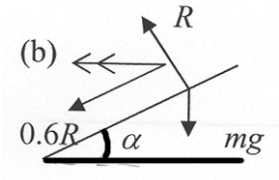


January 2008  
6679 Mechanics M3  
Mark Scheme

Question Number	Scheme	Marks
1.(a)	$T \text{ or } \frac{\lambda \times e}{l} = mg \quad (\text{even } T=m \text{ is M1, A0, A0 sp case})$ $\frac{\lambda \times 0.16}{0.4} = 2g$	M1 A1
(b)	$\Rightarrow \lambda = 49 \text{ N} \quad \text{or } 5g$ <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Special case <math>T \sin \theta = mg</math>  giving <math>\theta = 30</math> is M1 A0 A0 unless  there is evidence that they think <math>\theta</math> is  with horizontal – then M1 A1 A0 </div> $R(\uparrow) \quad T \cos \theta = mg \text{ or } \cos \theta = \frac{mg}{T}$ $49 \cdot \frac{0.32}{0.4} \cdot \cos \theta = 19.6 \text{ or } 4g \cdot \cos \theta = 2g \text{ or } 2mg \cdot \cos \theta = mg \quad (\text{ft on their } \lambda)$ $\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ \quad (\text{or } \frac{\pi}{3} \text{ radians})$	A1 (3) M1 A1ft A1 (3) <b>6</b>
2.	<p>a)</p> $m 'a' = \pm \frac{16}{5x^2}, \text{ with acceleration in any form (e.g. } \frac{d^2x}{dt^2}, v \frac{dv}{dx}, \frac{dv}{dt} \text{ or } v \frac{dv}{dx} \text{ to obtain } k v \frac{dv}{dx} = \pm k' \frac{32}{x^2})$ <p>Uses <math>a = v \frac{dv}{dx}</math> to obtain <math>k v \frac{dv}{dx} = \pm k' \frac{32}{x^2}</math></p> <p>Separates variables, <math>k \int v dv = k' \int \frac{32}{x^2} dx</math></p> <p>Obtains <math>\frac{1}{2} v^2 = \mp \frac{32}{x} (+ C) \text{ or equivalent e.g. } \frac{0.1}{2} v^2 = - \frac{16}{5x} (+ C)</math></p> <p>Substituting <math>x = 2</math> if + used earlier or <math>-2</math> if – used in d.e.  <math>x = 2, v = \pm 8 \Rightarrow 32 = -16 + C \Rightarrow C = 48</math> (or value appropriate to their correct equation)</p> <p><math>v = 0 \Rightarrow \frac{32}{x} = 48 \Rightarrow x = \frac{2}{3} \text{ m} \quad (\text{N.B. } -\frac{2}{3} \text{ is not acceptable for final answer})</math></p>	B1 M1 dM1 A1 M1 A1 M1 A1 cao <b>8</b>
	<p>N.B. <math>\frac{d}{dx} (\frac{1}{2} m v^2) = \frac{16}{5x^2}</math>, is also a valid approach.</p> <p>Last two method marks are independent of earlier marks and of each other</p>	

Question Number	Scheme	Marks
3.(a)	<div> <div> <div>Large cone</div> <div>small cone</div> <div>S</div> </div> <div> <div>Vol.</div> <div><math>\frac{1}{3}\pi(2r)^2(2h)</math></div> <div><math>\frac{1}{3}\pi r^2 h</math></div> <div><math>\frac{7}{3}\pi r^2 h</math> (accept ratios 8 : 1 : 7)</div> </div> <div> <div>C of M</div> <div><math>\frac{1}{2}h,</math></div> <div><math>\frac{5}{4}h</math></div> <div><math>\bar{x}</math></div> <div>(or equivalent)</div> </div> <div> <div><math>\frac{8}{3}\pi r^2 h \cdot \frac{1}{2}h - \frac{1}{3}\pi r^2 h \cdot \frac{5}{4}h = \frac{7}{3}\pi r^2 h \cdot \bar{x}</math></div> <div>or equivalent</div> </div> <div> <div><math>\rightarrow \bar{x} = \frac{11}{28}h</math></div> <div>*</div> </div> </div>	<div>B1</div> <div>B1, B1</div> <div>M1</div> <div>A1</div> <div>(5)</div>
(b)	<div> <div><math>\tan \theta = \frac{2r}{\bar{x}} = \frac{2r}{\frac{11}{28}h}, = \frac{2r}{\frac{11}{14}r} = \frac{28}{11}</math></div> <div><math>\theta \approx 68.6^\circ</math> or 1.20 radians</div> <div>(Special case – obtains complement by using <math>\tan \theta = \frac{2r}{\bar{x}}</math> giving <math>21.4^\circ</math> or .374 radians M1A0A0)</div> </div>	<div>M1, A1</div> <div>A1</div> <div>(3)</div> <div>8</div>
	<p>Centres of mass may be measured from another point ( e.g. centre of small circle, or vertex) The Method mark will then require a complete method (Moments and subtraction) to give required value for <math>\bar{x}</math> ). However B marks can be awarded for correct values if the candidate makes the working clear.</p>	

4. (a)	<p>Energy equation with at least three terms, including K.E term</p> $\frac{1}{2}mV^2 + ..$ $+ .. \frac{1}{2} \cdot \frac{2mg}{a} \cdot \frac{a^2}{16}, +mg \cdot \frac{1}{2}a \cdot \sin 30, = \frac{1}{2} \cdot \frac{2mg}{a} \cdot \frac{9a^2}{16}$ $\Rightarrow V = \sqrt{\frac{ga}{2}}$	<p>M1</p> <p>A1, A1, A1</p> <p>dM1 A1 (6)</p>
(b)	<p>Using point where velocity is zero and point where string becomes slack:</p> $\frac{1}{2}mw^2 =$ $\frac{1}{2} \cdot \frac{2mg}{a} \cdot \frac{9a^2}{16}, -mg \cdot \frac{3a}{4} \cdot \sin 30$ $\Rightarrow w = \sqrt{\frac{3ag}{8}}$ <p>Alternative (using point of projection and point where string becomes slack):</p> $\frac{1}{2}mw^2 - \frac{1}{2}mV_1^2, = \frac{mga}{16} - \frac{mga}{8}$ $\text{So } w = \sqrt{\frac{3ag}{8}}$	<p>M1</p> <p>A1, A1</p> <p>A1 (4)</p> <p>M1,A1 A1</p> <p>A1</p> <p><b>10</b></p>
	<p>In part (a) DM1 requires EE, PE and KE to have been included in the energy equation.</p> <p>If sign errors lead to <math>V^2 = -\frac{ga}{2}</math>, the last two marks are M0 A0</p> <p>In parts (a) and (b) A marks need to have the correct signs</p> <p>In part (b) for M1 need <b>one</b> KE term in energy equation of at least <b>3 terms</b> with distance <math>\frac{3a}{4}</math> to indicate first method, and <b>two</b> KE terms in energy equation of at least <b>4 terms</b> with distance <math>\frac{a}{4}</math> to indicate second method.</p> <p>SHM approach in part (b). (<b>Condone this method only if SHM is proved</b>)</p> <p>Using <math>v^2 = \omega^2(a^2 - x^2)</math> with <math>\omega^2 = \frac{2g}{a}</math> and <math>x = \pm \frac{a}{4}</math>.</p> <p>Using '<math>a</math>' = <math>\frac{a}{2}</math> to give <math>w = \sqrt{\frac{3ag}{8}}</math>.</p>	<p>M1 A1 A1</p> <p>A1</p>

5.(a)	 $\frac{mv^2}{r} = \mu N, = \mu mg$ $\mu = \frac{v^2}{rg} = \frac{21^2}{75 \times 9.8} = 0.6 \quad *$	M1, A1  A1 (3)
(b)	 $R(\uparrow) \quad R \cos \alpha, \mp 0.6R \sin \alpha = mg$ $\Rightarrow R \left( \frac{4}{5} - \frac{3}{5} \cdot \frac{3}{5} \right) = mg \Rightarrow R = \frac{25mg}{11}$	M1, A1, A1  A1 (4)
(c)	$R(\leftarrow) \quad R \sin \alpha, \pm 0.6R \cos \alpha = \frac{mv^2}{r}$ $v \approx 32.5 \text{ m s}^{-1}$	M1, A1, A1  dM1 A1cao (5) <b>12</b>
	<p>In part (b) M1 needs three terms of which one is mg If <math>\cos \alpha</math> and <math>\sin \alpha</math> are interchanged in equation this is awarded M1 A0 A1</p> <p>In part (c) M1 needs three terms of which one is <math>\frac{mv^2}{r}</math> or <math>mr\omega^2</math> If <math>\cos \alpha</math> and <math>\sin \alpha</math> are interchanged in equation this is also awarded M1 A0 A1</p> <p>If they <b>resolve along the plane and perpendicular to the plane</b> in part (b), then attempt at  <math display="block">R - mg \cos \alpha = \frac{mv^2}{r} \sin \alpha, \text{ and } 0.6R + mg \sin \alpha = \frac{mv^2}{r} \cos \alpha</math> and attempt to eliminate <math>v</math></p> <p>Two correct equations Correct work to solve simultaneous equations Answer</p> <p>In part (c) Substitute R into one of the equations Substitutes into a correct equation (earning accuracy marks in part (b)) Uses <math>R = \frac{25mg}{11}</math> (or <math>\frac{25mg}{29}</math>) Obtain <math>v = 32.5</math></p>	M1 A1 A1 A1 (4)  M1 A1 A1 M1A1 (5)

6.(a)	<p>Energy equation with two terms on RHS, <math>\frac{1}{2}mv^2 = \frac{1}{2}m \cdot \frac{5ga}{2} + mga \sin \theta</math></p> <p><math>\Rightarrow v^2 = \frac{ga}{2}(5 + 4 \sin \theta)</math> *</p>	<p>M1, A1</p> <p>A1 cso (3)</p>
(b)	<p>R(\ string) <math>T - mg \sin \theta = \frac{mv^2}{a}</math> (3 terms)</p> <p><math>\Rightarrow T = \frac{mg}{2}(5 + 6 \sin \theta)</math> o.e.</p>	<p>M1 A1</p> <p>A1 (3)</p>
(c)	<p><math>T = 0 \Rightarrow \sin \theta, = -\frac{5}{6}</math></p> <p>Has a solution, so string slack when <math>\alpha \approx 236(.4)^\circ</math> or 4.13 radians</p>	<p>M1, A1</p> <p>A1 (3)</p>
(d)	<p>At top of small circle, <math>\frac{1}{2}mv^2 = \frac{1}{2}m \cdot \frac{5ga}{2} - \frac{mga}{2}</math> (M1 for energy equation with 3 terms)</p> <p><math>\Rightarrow v^2 = \frac{3}{2}ga = 14.7a</math></p> <p>Resolving and using Force = <math>\frac{mv^2}{r}</math>, <math>T + mg = m \cdot \frac{\frac{3}{2}ga}{\frac{1}{2}a}</math> (M1 needs three terms, but any v)</p> <p><math>\Rightarrow T = 2mg</math></p>	<p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>A1 (6)</p> <p><b>15</b></p>
	<p>Use of <math>v^2 = u^2 + 2gh</math> is M0 in part (a)</p>	

7.(a)	<p>(Measuring <math>x</math> from <math>E</math>) <math>2\ddot{x} = 2g - 98(x + 0.2)</math>, and so <math>\ddot{x} = -49x</math></p> <p>SHM period with <math>\omega^2 = 49</math> so <math>T = \frac{2\pi}{7}</math></p>	M1 A1, A1 d M1 A1cso (5)
(b)	Max. acceleration = $49 \times \text{max. } x = 49 \times 0.4 = 19.6 \text{ m s}^{-2}$	B1 (1)
(c)	<p>String slack when <math>x = -0.2</math>: <math>v^2 = 49(0.4^2 - 0.2^2)</math></p> <p><math>\Rightarrow v \approx 2.42 \text{ m s}^{-1} = \frac{7\sqrt{3}}{5}</math></p>	M1 A1 A1 (3)
(d)	<p>Uses <math>x = a \cos \omega t</math> or use <math>x = a \sin \omega t</math> but not with <math>x = 0</math> or <math>\pm a</math></p> <p>Attempt complete method for finding time when string goes slack <math>-0.2 = 0.4 \cos 7t \Rightarrow \cos 7t = -\frac{1}{2}</math></p> <p><math>t = \frac{2\pi}{21} \approx 0.299 \text{ s}</math></p> <p>Time when string is slack = <math>\frac{(2) \times 2.42}{g} = \frac{2\sqrt{3}}{7} \approx 0.495 \text{ s}</math> (2 needed for A )</p> <p>Total time = <math>2 \times 0.299 + 0.495 \approx 1.09 \text{ s}</math></p>	<p>M1</p> <p>dM1 A1</p> <p>A1</p> <p>M1 A1ft</p> <p>A1 (7)</p> <p><b>16</b></p>
(a)	<p>DM1 requires the minus sign.</p> <p>Special case</p> <p><math>2\ddot{x} = 2g - 98x</math> is M1A1A0M0A0 <math>2\ddot{x} = -98x</math> is M0A0A0M0A0</p>	
(b)	No use of $\ddot{x}$ , just $a$ is M1 A0,A0 then M1 A0 if otherwise correct.	
(c)	Quoted results are not acceptable.	
(d)	<p>Answer must be positive and evaluated for B1</p> <p>M1 – Use correct formula with their <math>\omega</math>, <math>a</math> and <math>x</math> but <b>not</b> <math>x = 0</math>.</p> <p>A1 Correct values but allow <math>x = +0.2</math></p> <p><b>Alternative</b></p> <p>It is possible to use energy instead to do this part</p> <p><math>\frac{1}{2}mv^2 + mg \times 0.6 = \frac{\lambda \times 0.6^2}{2l}</math> M1 A1</p> <p>If they use <math>x = a \sin \omega t</math> with <math>x = \pm 0.2</math> and add <math>\frac{\pi}{7}</math> or <math>\frac{\pi}{14}</math> this is dM1, A1 if done correctly</p> <p>If they use <math>x = a \cos \omega t</math> with <math>x = -0.2</math> this is dM1, then A1 (as in scheme)</p> <p>If they use <math>x = a \cos \omega t</math> with <math>x = +0.2</math> this needs <i>their</i> <math>\frac{\pi}{7}</math> minus answer to reach dM1, then A1</p>	

GCE

Edexcel GCE

Mathematics

Mechanics 3 M3 (6679)

June 2008

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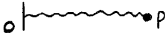

Mark Scheme (Final)

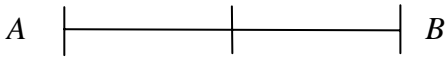
## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.



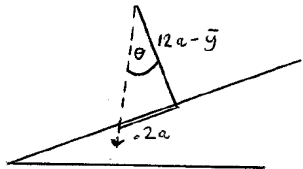


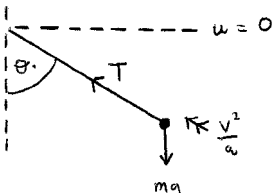
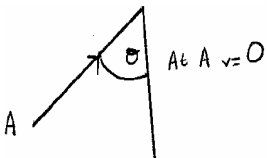
June 2008  
6679 Mechanics M3  
Mark Scheme

Question Number	Scheme	Marks
Q1(a)	 $\text{EPE stored} = \frac{1}{2} \frac{\lambda}{L} \left( \frac{1}{2} L \right)^2 \quad \left( = \frac{\lambda L}{8} \right)$ $\text{KE gained} = \frac{1}{2} m 2gL \quad (= mgL)$ $\text{EPE} = \text{KE} \Rightarrow \frac{\lambda L}{8} = mgL \quad \text{i.e. } \lambda = 8mg^*$	<p>B1</p> <p>B1</p> <p>M1A1cso</p> <p><b>(4)</b></p>
	<p>(b)</p>  $\text{EPE} = \text{GPE} + \text{KE}$ $\frac{1}{2} \frac{8mg}{L} \left( \frac{1}{2} L \right)^2 = \frac{8mgL}{8} = mg \frac{L}{2} + \frac{1}{2} m u^2$ $\frac{mgL}{2} = \frac{m}{2} u^2 \quad \therefore u = \sqrt{gL}$	<p>M1</p> <p>A1A1</p> <p>M1A1 <b>(5)</b></p> <p><b>9 Marks</b></p>

Question Number	Scheme	Marks
Q2 (a)	<p style="text-align: center;"><math>O</math></p> <p style="text-align: center;">  </p> $T = 3 = \frac{2\pi}{\omega} \quad \therefore \omega = \frac{2\pi}{3}$ $u^2 = \omega^2 (a^2 - x^2) \quad ; \quad a = 0.12 \quad , \quad u^2 = a^2 \omega^2, u = 0.12 \times \omega$ $= 0.251 \text{ ms}^{-1} \quad (0.25 \text{ m s}^{-1})$	<p>M1A1</p> <p>M1</p> <p>A1    <b>(4)</b></p>
(b)	<p>Time from <math>O \rightarrow A \rightarrow O = 1.5\text{s} \quad \therefore t = 0.5</math></p> $x = a \sin \omega t \quad \Rightarrow OP = 0.12 \sin\left(\frac{\pi}{3}\right)$ <p>Distance from <math>B</math> is <math>0.12 - OP = 0.12 - 0.104\dots = 0.016\text{m}</math></p>	<p>B1</p> <p>M1A1</p> <p>M1A1 <b>(5)</b></p>
(c)	$v^2 = \omega^2 (a^2 - x^2)$ $v = \frac{2\pi}{3} \sqrt{0.12^2 - 0.104\dots^2} = \frac{2\pi}{3} \times 0.0598 = 0.13 \text{ ms}^{-1}$	<p>M1</p> <p>A1    <b>(2)</b></p> <p><b>11 Marks</b></p>

Question Number	Scheme	Marks
<p>Q3 (a)</p>	<div data-bbox="228 241 430 465"> </div> <div data-bbox="288 577 491 835"> </div> <div data-bbox="632 510 1110 600"> <math display="block">\begin{aligned} \uparrow \quad T \cos \theta + N &amp;= Mg &amp; (1) \\ \rightarrow \quad T \sin \theta &amp;= mr\omega^2 &amp; (2) \end{aligned}</math> </div> <div data-bbox="595 689 1031 757"> <math display="block">\sin \theta = \frac{r}{l} \quad \text{from (2)} \quad T = ml\omega^2</math> </div> <div data-bbox="603 801 1069 842"> <math display="block">\text{sub into (1)} \quad ml \cos \theta \omega^2 + N = mg</math> </div> <div data-bbox="754 887 962 925"> <math display="block">N = mg - mh\omega^2</math> </div> <div data-bbox="568 976 1193 1043"> <math display="block">\text{Since in contact with table } N \geq 0 \quad \therefore \omega^2 \leq \frac{g}{h}</math> </div> <div data-bbox="76 1122 113 1160"> <p>(b)</p> </div> <div data-bbox="225 1200 683 1272"> <math display="block">r : h : l = 3 : 4 : 5 \quad \therefore \text{extension} = \frac{h}{4}</math> </div> <div data-bbox="225 1283 499 1361"> <math display="block">T = \frac{2mg}{h} \times \frac{h}{4} = \frac{mg}{2}</math> </div> <div data-bbox="225 1373 627 1451"> <math display="block">T = ml\omega^2 = \frac{5mh}{4}\omega^2 \quad \omega = \sqrt{\frac{2g}{5h}}</math> </div>	<div data-bbox="1361 465 1445 499">M1A1</div> <div data-bbox="1361 539 1445 573">M1A1</div> <div data-bbox="1361 797 1406 831">M1</div> <div data-bbox="1361 871 1401 904">A1</div> <div data-bbox="1361 976 1501 1010">M1A1 cso</div> <div data-bbox="1422 1050 1461 1084">(8)</div> <div data-bbox="1361 1234 1401 1267">B1</div> <div data-bbox="1361 1308 1445 1341">M1A1</div> <div data-bbox="1361 1420 1445 1453">M1A1</div> <div data-bbox="1422 1494 1461 1527">(5)</div> <div data-bbox="1361 1565 1493 1599">13 marks</div>

Question Number	Scheme	Marks
Q4 (a)	 $\text{Mass } a^3 \frac{2}{3} \pi \times: \quad \frac{216}{27} \quad \frac{8}{1} \quad \frac{208}{26}$ $\text{C of M from } O: \quad \frac{3}{8} \times 6a \quad \frac{3}{8} \times 2a \quad \bar{x}$ $\text{Moment: } 216 \times \frac{6a \times 3}{8} = 8 \times \frac{2a \times 3}{8} + 208\bar{x}$ $\bar{x} = \frac{480a}{208} = \frac{30a}{13} *$	M1A1 M1 M1 A1 cso <b>(5)</b>
(b)	 $\text{Mass } \pi a^3 \times: \quad \frac{416}{3} + 24 = \frac{488}{3}$ $\text{C of M: } \quad \frac{30}{13}a + 9a = \bar{y}$ $\text{Moments: } \quad 320a + 216a = \frac{488}{3}\bar{y}$ $\bar{y} = \frac{201}{61}a *$	B1 B1 M1 A1 cso <b>(4)</b>
(c)	 $\tan \theta = \frac{2a}{12a - \frac{201}{61}a}$ $\theta = 12.93 \dots$ <p>so critical angle = 12.93...      <math>\therefore</math> if <math>\theta = 12^\circ</math> it will <u>NOT</u> topple.</p>	M1 M1 A1 A1✓ <b>(4)</b> <b>13 marks</b>

Question Number	Scheme	Marks
Q5(a)	 <p>Energy <math>\frac{1}{2} mv^2 = mga \cos \theta</math>  <math>v^2 = 2ga \cos \theta</math></p> <p><math>F = ma \quad T - mg \cos \theta = \frac{mv^2}{a}</math></p> <p>Sub for <math>\frac{v^2}{a}</math>: <math>T = mg \cos \theta + 2mg \cos \theta</math>: <math>\theta = 60^\circ \therefore T = \frac{3}{2} mg</math></p>	<p>M1A1</p> <p>M1A1</p> <p>M1A1</p> <p>(6)</p>
(b)	<p>Speed of P before impact = <math>\sqrt{2ga}</math></p> <p>PCLM: <math>\begin{matrix} \rightarrow \sqrt{2ga} &amp; \rightarrow 0 &amp; \rightarrow u \\ \bullet &amp; \bullet &amp; \bullet \\ m &amp; 3m &amp; 4m \end{matrix} \quad \therefore u = \frac{\sqrt{2ga}}{4} = \sqrt{\frac{ga}{8}} *</math></p>	<p>B1</p> <p>M1A1cso</p> <p>(3)</p>
(c) (i)	<p>At A <math>v = 0</math> so conservation of energy gives:</p>  <p><math>\frac{1}{2} 4mu^2 = 4mga (1 - \cos \theta)</math></p> <p><math>\frac{ga}{16} = ga (1 - \cos \theta)</math></p> <p><math>\cos \theta = \frac{15}{16}, \theta = 20^\circ</math></p>	<p>M1A1</p> <p>M1</p> <p>A1</p>
(ii)	<p>At A <math>T = 4mg \cos \theta = \frac{15mg}{4}</math> (accept 3.75mg)</p>	<p>M1A1 (6)</p> <p>15 Marks</p>

Question Number	Scheme	Marks
Q6 (a)	$F = ma \Rightarrow \frac{3}{(x+1)^3} = 0.5a = 0.5 v \frac{dv}{dx}$ $\int \frac{3}{(x+1)^3} dx = 0.5 \int v dv$ <p style="text-align: right;">Separate and ∫</p> $-\frac{3}{2(x+1)^2} = \frac{1}{4} v^2 (+ c)$ $x = 0, v = 0 \Rightarrow c' = -\frac{3}{2} \quad \therefore v^2 = 6 \left( 1 - \frac{1}{(x+1)^2} \right) *$	<p>M1A1</p> <p>M1</p> <p>A1</p> <p>M1A1 cso (6)</p>
(b)	$\forall x \quad v^2 < 6 \quad \therefore v < \sqrt{6} \quad (\because (x+1)^2 \text{ always } > 0)$	<p>B1 (1)</p>
(c)	$v = \frac{dx}{dt} = \frac{\sqrt{6}\sqrt{(x+1)^2 - 1}}{x+1}$ $\int \frac{x+1}{\sqrt{(x+1)^2 - 1}} dx = \sqrt{6} \int dt$ $\sqrt{(x+1)^2 - 1} = \sqrt{6} t + c'$ $t = 0, x = 0 \Rightarrow c' = 0$ $t = 2 \Rightarrow (x+1)^2 - 1 = (2\sqrt{6})^2$ $(x+1)^2 = 25 \Rightarrow x = 4 \quad (c' \text{ need not have been found})$	<p>M1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1 cao</p> <p>(7)</p> <p><b>14 Marks</b></p>

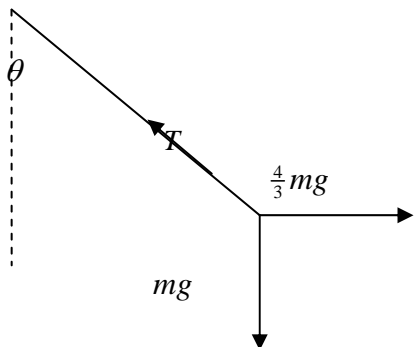
# Mark Scheme (Results)

## January 2009

GCE

GCE Mathematics (6679/01)

January 2009  
6679 Mechanics M3  
Mark Scheme

Question Number	Scheme	Marks
1	<p>N2L</p> $3a = -\left(9 + \frac{15}{(t+1)^2}\right)$ $3v = -9t + \frac{15}{t+1} (+A)$ $v = 0, t = 4 \Rightarrow 0 = -36 + 3 + A \Rightarrow A = 33$ $v = -3t + \frac{5}{t+1} + 11$ $t = 0 \Rightarrow v = 16$	<p>B1</p> <p>M1 A1ft</p> <p>M1 A1</p> <p>M1 A1 (7) [7]</p>
2	<div style="text-align: center;">  </div> <p>(a)</p> $\begin{aligned} (\leftarrow) \quad T \sin \theta &= \frac{4}{3}mg \\ (\uparrow) \quad T \cos \theta &= mg \\ T^2 &= \left(\frac{4}{3}mg\right)^2 + (mg)^2 \\ \text{Leading to } T &= \frac{5}{3}mg \end{aligned}$ <p>(b)</p> $\begin{aligned} \text{HL } T &= \frac{\lambda x}{a} \Rightarrow \frac{5}{3}mg = \frac{3mge}{a} && \text{ft their } T \\ e &= \frac{5}{9}a \\ E &= \frac{\lambda x^2}{2a} = \frac{3mg}{2a} \times \left(\frac{5}{9}a\right)^2 = \frac{25}{54}mga \end{aligned}$	<p>M1 A1</p> <p>A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1 A1ft</p> <p>M1 A1 (4) [9]</p>



Question Number	Scheme	Marks
3	$\omega = \frac{80 \times 2\pi}{60} \text{ rad s}^{-1} \left( = \frac{8\pi}{3} \approx 8.377 \dots \right)$ <p>Accept <math>v = \frac{16\pi}{75} \approx 0.67 \text{ ms}^{-1}</math> as equivalent</p> $(\uparrow) R = mg$ <p>For least value of <math>\mu</math> <math>(\leftarrow) \mu mg = mr\omega^2</math></p> $\mu = \frac{0.08}{9.8} \times \left( \frac{8\pi}{3} \right)^2 \approx 0.57 \quad \text{accept 0.573}$	<p>B1</p> <p>B1</p> <p>M1 A1=A1</p> <p>M1 A1 (7)</p> <p>[7]</p>
4	<p>(a)</p> $a = 8$ $T = \frac{25}{2} = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{4\pi}{25} (\approx 0.502 \dots)$ $v^2 = \omega^2 (a^2 - x^2) \Rightarrow v^2 = \left( \frac{4\pi}{25} \right)^2 (8^2 - 3^2) \quad \text{ft their } a, \omega$ $v = \frac{4\pi}{25} \sqrt{55} \approx 3.7 \text{ (m h}^{-1}\text{)} \quad \text{awrt 3.7}$ <p>(b)</p> $x = a \cos \omega t \Rightarrow 3 = 8 \cos \left( \frac{4\pi}{25} t \right) \quad \text{ft their } a, \omega$ $t \approx 2.3602 \dots$ <p>time is 12 22</p>	<p>B1</p> <p>M1 A1</p> <p>M1 A1ft</p> <p>M1 A1 (7)</p> <p>M1 A1ft</p> <p>M1 A1 (4)</p> <p>[11]</p>

Question Number	Scheme	Marks
5	<p>(a) Let <math>x</math> be the distance from the initial position of <math>B</math> to <math>C</math>  GPE lost = EPE gained  <math display="block">mgx \sin 30^\circ = \frac{6mgx^2}{2a}</math> Leading to <math>x = \frac{a}{6}</math>  <math display="block">AC = \frac{7a}{6}</math></p> <p>(b) The greatest speed is attained when the acceleration of <math>B</math> is zero, that is where the forces on <math>B</math> are equal.  <math display="block">(T) \quad T = mg \sin 30^\circ = \frac{6mge}{a}</math> <math display="block">e = \frac{a}{12}</math> CE <math display="block">\frac{1}{2}mv^2 + \frac{6mg}{2a} \left(\frac{a}{12}\right)^2 = mg \frac{a}{12} \sin 30^\circ</math> Leading to <math display="block">v = \sqrt{\left(\frac{ga}{24}\right)} = \frac{\sqrt{6ga}}{12}</math></p> <p><i>Alternative approaches to (b) are considered on the next page.</i></p>	<p>M1 A1=A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1</p> <p>A1</p> <p>M1 A1=A1</p> <p>M1 A1 (7)</p> <p>[12]</p>

Question Number	Scheme	Marks
5	<p><i>Alternative approach to (b) using calculus with energy.</i></p> <p>Let distance moved by <math>B</math> be <math>x</math></p> <p>CE <math>\frac{1}{2}mv^2 + \frac{6mg}{2a}x^2 = mgx \sin 30^\circ</math></p> $v^2 = gx - \frac{6g}{a}x^2$ <p>For maximum <math>v</math> <math>\frac{d}{dx}(v^2) = 2v \frac{dv}{dx} = g - \frac{12g}{a}x = 0</math></p> $x = \frac{a}{12}$ $v^2 = g\left(\frac{a}{12}\right) - \frac{6g}{a}\left(\frac{a}{12}\right)^2 = \frac{ga}{24}$ $v = \sqrt{\left(\frac{ga}{24}\right)}$	<p>M1 A1=A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p>
	<p><i>Alternative approach to (b) using calculus with Newton's second law.</i></p> <p>As before, the centre of the oscillation is when extension is <math>\frac{a}{12}</math></p> <p>N2L <math>mg \sin 30^\circ - T = m\ddot{x}</math></p> $\frac{1}{2}mg - \frac{6mg\left(\frac{a}{12} + x\right)}{a} = m\ddot{x}$ $\ddot{x} = -\frac{6g}{a}x \Rightarrow \omega^2 = \frac{6g}{a}$ $v_{\max} = \omega a = \sqrt{\left(\frac{6g}{a}\right)} \times \frac{a}{12} = \sqrt{\left(\frac{ga}{24}\right)}$	<p>M1 A1</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1 (7)</p>

Question Number	Scheme	Marks
<p>6 (a)</p> <p>(b)</p>	$\int y^2 dx = \int (4 - x^2)^2 dx = \int (16 - 8x^2 + x^4) dx$ $= 16x - \frac{8x^3}{3} + \frac{x^5}{5}$ $\left[ 16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2 = \frac{256}{15}$ $\int xy^2 dx = \int x(4 - x^2)^2 dx = \int (16x - 8x^3 + x^5) dx$ $= 8x^2 - 2x^4 + \frac{x^6}{6}$ $\left[ 8x^2 - 2x^4 + \frac{x^6}{6} \right]_0^2 = \frac{32}{3}$ $\bar{x} = \frac{\int xy^2 dx}{\int y^2 dx} = \frac{32}{3} \times \frac{15}{216} = \frac{5}{8} *$ $A \times \bar{x} = (\pi r^2 l) \times \frac{l}{2}$ $\frac{256}{15} \pi \times \frac{5}{8} = \pi \times 16l \times \frac{l}{2}$ <p>Leading to <math>l = \frac{2\sqrt{3}}{3}</math>      accept exact equivalents or awrt 1.15</p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1A1</p> <p>M1 A1 (10)</p> <p>M1</p> <p>A1 ft</p> <p>M1 A1 (4)</p> <p>[14]</p>

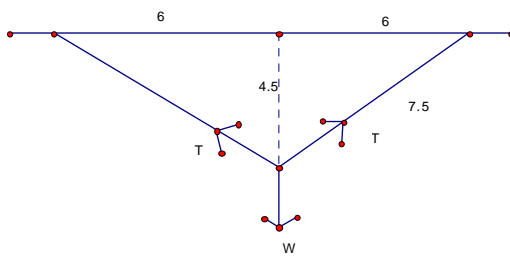
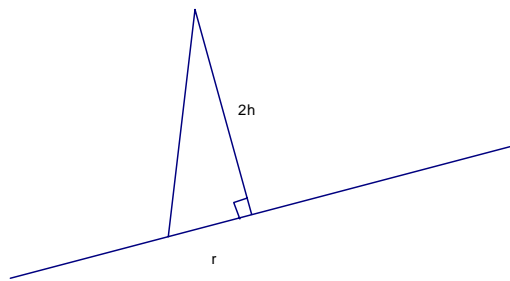
Question Number	Scheme	Marks
7 (a)	<p>Let speed at C be <math>u</math></p> <p>CE <math>\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mga(1 - \cos \theta)</math></p> $u^2 = \frac{9ga}{4} - 2ga \cos \theta$ $mg \cos \theta (+R) = \frac{mu^2}{a}$ $mg \cos \theta = \frac{9mg}{4} - 2mg \cos \theta \quad \text{eliminating } u$ <p>Leading to <math>\cos \theta = \frac{3}{4} *</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 (7)</p>
(b)	<p>At C <math>u^2 = \frac{9ga}{4} - 2ga \times \frac{3}{4} = \frac{3}{4}ga</math></p> <p>(<math>\rightarrow</math>) <math>u_x = u \cos \theta = \sqrt{\left(\frac{3ga}{4}\right) \times \frac{3}{4}} = \sqrt{\left(\frac{27ga}{64}\right)} = 2.033\sqrt{a}</math></p> <p>(<math>\downarrow</math>) <math>u_y = u \sin \theta = \sqrt{\left(\frac{3ga}{4}\right) \times \frac{\sqrt{7}}{4}} = \sqrt{\left(\frac{21ga}{64}\right)} = 1.792\sqrt{a}</math></p> $v_y^2 = u_y^2 + 2gh \Rightarrow v_y^2 = \frac{21}{64}ga + 2g \times \frac{7}{4}a = \frac{245}{64}ga$ $\tan \psi = \frac{v_y}{u_x} = \sqrt{\left(\frac{245}{27}\right)} \approx 3.012 \dots$ <p><math>\psi \approx 72^\circ</math> awrt <math>72^\circ</math></p> <p>Or <math>1.3^\circ</math> (1.2502°) awrt <math>1.3^\circ</math></p>	<p>B1</p> <p>M1 A1ft</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (8)</p> <p>[15]</p>
<p><i>Alternative for the last five marks</i></p> <p>Let speed at P be <math>v</math>.</p> <p>CE <math>\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mg \times 2a</math> or equivalent</p> $v^2 = \frac{17mga}{4}$ $\cos \psi = \frac{u_x}{v} = \sqrt{\left(\frac{27}{64} \times \frac{4}{17}\right)} = \sqrt{\left(\frac{27}{272}\right)} \approx 0.315$ <p><math>\psi \approx 72^\circ</math> awrt <math>72^\circ</math></p> <p><i>Note: The time of flight from C to P is <math>\frac{\sqrt{235} - \sqrt{21}}{8} \sqrt{\left(\frac{a}{g}\right)} \approx 1.38373 \sqrt{\left(\frac{a}{g}\right)}</math></i></p>		<p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p>

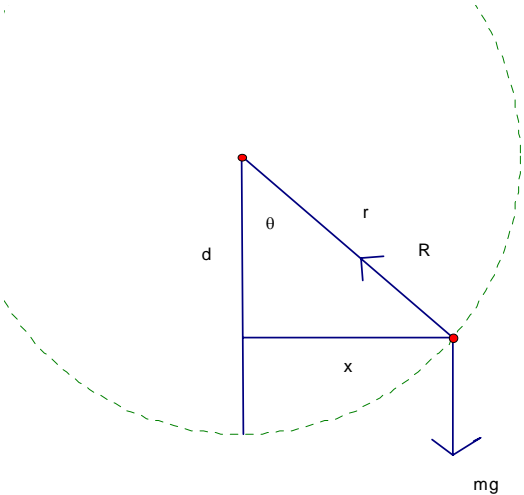
# Mark Scheme (Results) Summer 2009

GCE

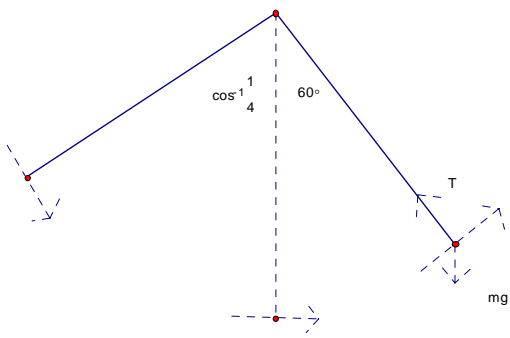
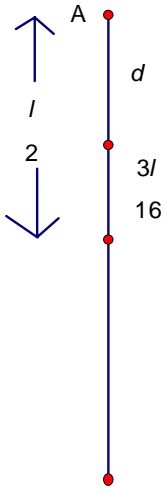
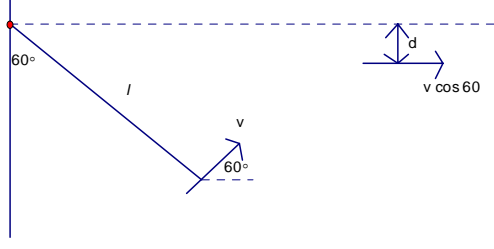
GCE Mathematics (6679/01)

June 2009  
6679 Mechanics M3  
Mark Scheme

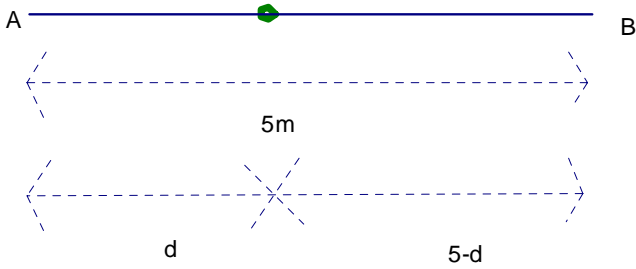
Question Number	Scheme			Marks													
Q1	(a)		Resolving vertically: $2T \cos \theta = W$  Hooke's Law: $T = \frac{80 \times 3.5}{4}$ $W = 84\text{N}$	M1A2,1,0  M1A1  A1													
	(b)	$\text{EPE} = 2 \times \frac{80 \times 3.5^2}{2 \times 4}, = 245 \text{ (or awrt 245)}$  (alternative $\frac{80 \times 7^2}{16} = 245$ )		M1A1ft,A1   [9]													
Q2	(a)	<table><tr><td>Object</td><td>Mass</td><td>c of m above base</td></tr><tr><td>Cone</td><td><math>m</math></td><td><math>2h+3h</math></td></tr><tr><td>Base</td><td><math>3m</math></td><td><math>h</math></td></tr><tr><td>Marker</td><td><math>4m</math></td><td><math>d</math></td></tr></table> $m \times 5h + 3m \times h = 4m \times d$  $d = 2h$	Object	Mass	c of m above base	Cone	$m$	$2h+3h$	Base	$3m$	$h$	Marker	$4m$	$d$	B1(ratio masses) B1(distances)		
Object	Mass	c of m above base															
Cone	$m$	$2h+3h$															
Base	$3m$	$h$															
Marker	$4m$	$d$															
	(b)		$\frac{r}{d} = \frac{1}{12}$  $6r = h$	M1A1ft  A1  M1A1ft   A1  [8]													

Question Number	Scheme	Marks
Q3 (a)		M1 A1
(b)	$\leftrightarrow R \sin \theta = m x \omega^2$ $R \times \frac{x}{r} = m x \times \frac{3g}{2r}$ $R = \frac{3mg}{2}$ $\updownarrow R \cos \theta = mg$ $\frac{3mg}{2} \times \frac{d}{r} = mg$ $d = \frac{2}{3}r$	M1 A1 M1 A1
		[8]
Q4 (a)	$\text{Volume} = \int_{\frac{1}{4}}^1 \pi y^2 dx = \int_{\frac{1}{4}}^1 \pi \frac{1}{x^4} dx$ $= \left[ \pi \times \frac{-1}{3x^3} \right]_{\frac{1}{4}}^1$ $= \pi \left( \frac{-1}{3} + \frac{64}{3} \right) = 21\pi \quad *$	M1A1 A1ft A1
(b)	$21\pi \bar{x} = \rho \int \pi y^2 x dx = \rho \int \pi \frac{1}{x^4} x dx$ $21\pi \bar{x} = \pi \left[ \frac{-1}{2x^2} \right]_{\frac{1}{4}}^1$ $\bar{x} = \frac{1}{21} \left( \frac{-1}{2} + \frac{16}{2} \right) = \frac{5}{14} \quad \text{or awrt } 0.36$ $\bar{y} = 0 \text{ by symmetry}$	M1A1 A1ft A1 B1
		[9]



Question Number	Scheme	Marks
Q5 (a)	 <p>Energy:</p> $\left(\frac{1}{2}mv^2 + \right)mgl\left(\cos\theta - \frac{1}{4}\right) = \frac{1}{2}mv^2$ <p>Resolving:</p> $T - mg\cos\theta = \frac{mv^2}{l}$ <p>Eliminate <math>v^2</math>:</p> $T = mg\cos\theta + \frac{1}{l}\left(2mgl\left(\cos\theta - \frac{1}{4}\right)\right)$ $T = 3mg\cos\theta - \frac{mg}{2} *$	M1A1 M1A1 M1 A1
(b)	 <p> <math>\theta = 60^\circ \Rightarrow mv^2 = 2mgl\left(\frac{1}{2} - \frac{1}{4}\right)</math>  <math>\Rightarrow v^2 = \frac{gl}{2}</math> </p> <p>vertical motion under gravity:</p> $\uparrow 0 = (v\cos 30^\circ)^2 - 2gs$ $0 = \frac{gl}{2} \times \frac{3}{4} - 2gs \Rightarrow s = \frac{3l}{16}$ <p>Distance below A = <math>\frac{l}{2} - \frac{3l}{16} = \frac{5l}{16}</math></p>	M1 M1 A1 M1A1  [11]
Alternative for end of (b) using energy	 <p> <math>\frac{1}{2}mv^2 - mgl\cos 60 = \frac{1}{2}m(v\cos 60)^2 - mgd</math>  <math>\frac{gl}{4} - \frac{gl}{2} = \frac{gl}{4} \times \frac{1}{4} - gd</math>  <math>d = \frac{1-4+8}{16}l = \frac{5l}{16}</math> </p>	M1A1 M1 A1

Question Number	Scheme	Marks
Q6 (a)	<p>At max v, driving force = resistance</p> $\text{Driving force} = \frac{80}{v}$ $\Rightarrow \frac{80}{20} = k \times 20^2 \Rightarrow k = \frac{1}{100}$ $F = ma \Rightarrow 100a = \frac{80}{v} - kv^2 \quad (= \frac{8000 - v^3}{100v})$ $\ast \Rightarrow v \frac{dv}{dx} = \frac{8000 - v^3}{10000v} \ast$	<p>B1</p> <p>M1A1</p> <p>M1</p> <p>A1</p>
(b)	$\int_4^8 \frac{10000v^2}{8000 - v^3} dv = \int_0^D 1 dx$ $D = \left[ -\frac{10000}{3} \ln 8000 - v^3  \right]_4^8$ $= \left( -\frac{10000}{3} \ln \frac{7488}{7936} \right) = 193.7 \dots \approx 194 \text{ m (accept 190)}$	<p>M1A1</p> <p>A1</p> <p>M1</p> <p>A1</p>
(c)	$\frac{dv}{dt} = \frac{8000 - v^3}{10000v} \Rightarrow \int_0^T 1 dt = \int_4^8 \frac{10000v}{8000 - v^3} dv$ $\Rightarrow T \approx \frac{1}{2} \times 2 \times 10000 \times \left\{ \frac{4}{7936} + \frac{2 \times 6}{7784} + \frac{8}{7488} \right\}$ $\Rightarrow T (= 31.1409 \dots) \approx 31$	<p>M1A1</p> <p>M1</p> <p>A1</p> <p>[14]</p>

Question Number	Scheme	Marks
Q7 (a)	<p>mod=16 a=2</p> <p>mod=12 a=1</p>  <p>Hooke's law: Equilibrium <math>\Rightarrow \frac{16(d-2)}{2} = \frac{12(4-d)}{1}</math>  <math>\Rightarrow d = 3.2</math>  so extensions are 1.2m and 0.8m.</p> <p>(b) If the particle is displaced distance <math>x</math> towards <b>B</b> then  <math>-m\ddot{x} = \frac{16(1.2+x)}{2} - \frac{12(0.8-x)}{1} (= 20x)</math>  <math>\Rightarrow \ddot{x} = -40x</math> or <math>\ddot{x} = -\frac{20}{m}(\Rightarrow \text{SHM})</math></p> <p>(c) <math>T = \frac{2\pi}{\sqrt{40}}</math>  <math>a = \frac{\sqrt{10}}{\text{their } \omega}</math>  <math>x = a \sin \omega t</math> their <math>a</math>, their <math>\omega</math>  <math>\frac{1}{4} = \frac{1}{2} \sin \sqrt{40}t</math>  <math>\sqrt{40}t = \frac{\pi}{6} (\Rightarrow t = \frac{\pi}{6\sqrt{40}})</math>  Proportion <math>\frac{4t}{T} = \frac{4\pi}{6\sqrt{40}} \times \frac{\sqrt{40}}{2\pi} = \frac{1}{3}</math></p>	<p>M1A1A1</p> <p>A1 A1</p> <p>M1A1ft A1ft</p> <p>A1</p> <p>B1ft</p> <p>B1ft</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1A1</p> <p>[16]</p>

# Mark Scheme (Results) January 2010

GCE

Mechanics M3 (6679)

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January 2010

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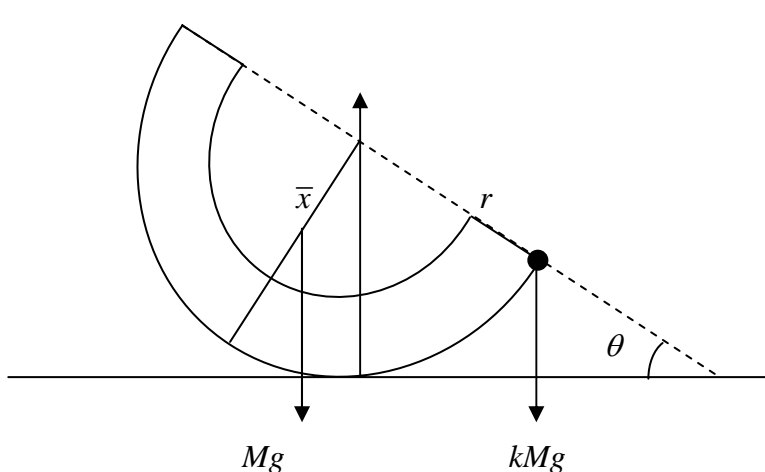
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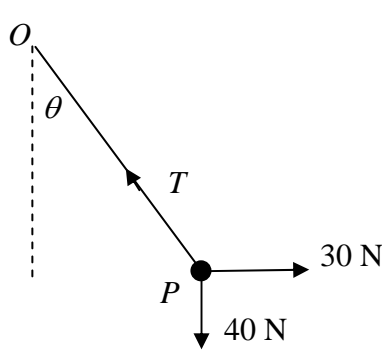
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6679 Mechanics M3  
Mark Scheme

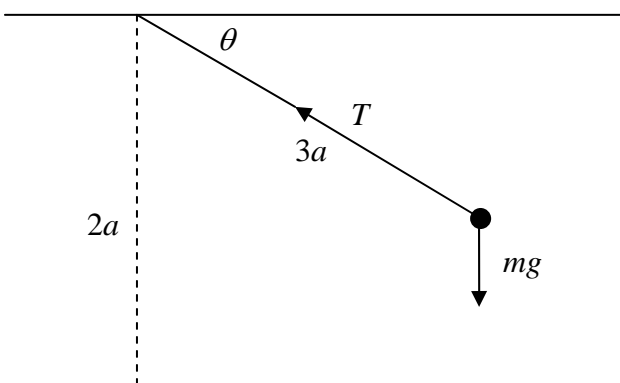
Question Number	Scheme	Marks
Q1.	$0.5a = 4 + \cos(\pi t)$ <p>Integrating <math>0.5v = 4t + \frac{\sin(\pi t)}{\pi} (+ C)</math></p> <p>Using boundary values  <math>3 = 4 + C \Rightarrow C = -1</math></p> <p>When <math>t = 1.5</math></p> $0.5v = 6 - \frac{1}{\pi} - 1$ $v \approx 9.36 \text{ (m s}^{-1}\text{)}$	<p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p> <p>[7]</p>

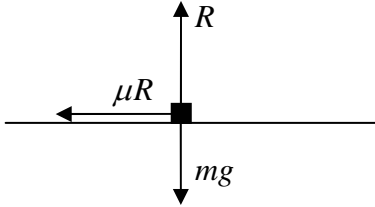
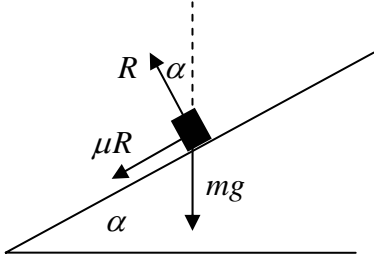
Question Number	Scheme	Marks
Q2.	<p>(a) <math>\frac{2\pi}{\omega} = 2.4 \Rightarrow \omega = \frac{5\pi}{6} (\approx 2.62)</math></p> <p><math>x = 0, t = 0 \Rightarrow x = a \sin \omega t</math></p> <p>when <math>t = 0.4</math>, <math>x = a \sin\left(\frac{5\pi}{6} \times 0.4\right) \quad \left(= \frac{\sqrt{3}}{2} a\right)</math></p> <p><math>v^2 = \omega^2 (a^2 - x^2) \Rightarrow 16 = \frac{25\pi^2}{36} \left(a^2 - \frac{3a^2}{4}\right) \Rightarrow a = \frac{48}{5\pi} (\approx 3.06)</math></p> <p><math>v_{\max} = a\omega = 8</math> (or awrt 8.0 if decimals used earlier) cao</p> <p>(b) <math>\ddot{x}_{\max} = a\omega^2 = \frac{20\pi}{3}</math> awrt 21</p>	<p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1 (7)</p> <p>M1 A1 (2)</p> <p>[9]</p>
	<p>Alternative in (a)</p> <p>(a) <math>\frac{2\pi}{\omega} = 2.4 \Rightarrow \omega = \frac{5\pi}{6}</math></p> <p><math>x = 0, t = 0 \Rightarrow x = a \sin \omega t</math></p> <p><math>\dot{x} = a\omega \cos \omega t</math></p> <p><math>4 = a\omega \cos\left(\frac{5\pi}{6} \times 0.4\right)</math></p> <p><math>a = \frac{48}{5\pi} (\approx 3.06)</math> or <math>a\omega = 8</math></p> <p><math>v_{\max} = a\omega = 8</math></p>	<p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1 A1 (7)</p>

Question Number	Scheme	Marks															
Q3.	<p>(a)</p> <table> <tr> <td></td> <td><math>s</math></td> <td><math>B</math></td> <td><math>S</math></td> <td></td> </tr> <tr> <td>Mass ratios</td> <td>8</td> <td>19</td> <td>27</td> <td>anything in correct ratio</td> </tr> <tr> <td><math>\bar{x}</math></td> <td><math>\frac{3}{8} \times \frac{2}{3} r</math></td> <td><math>\bar{x}</math></td> <td><math>\frac{3}{8} r</math></td> <td></td> </tr> </table> $8 \times \frac{1}{4} r + 19 \bar{x} = 27 \times \frac{3}{8} r$ $\bar{x} = \frac{65}{152} r \quad *$ <p>(b)</p>  $Mg \times \bar{x} \sin \theta = kMg \times r \cos \theta$ <p>leading to <math>k = \frac{13}{38}</math></p>		$s$	$B$	$S$		Mass ratios	8	19	27	anything in correct ratio	$\bar{x}$	$\frac{3}{8} \times \frac{2}{3} r$	$\bar{x}$	$\frac{3}{8} r$		<p>B1</p> <p>B1</p> <p>M1 A1ft</p> <p>A1 (5)</p> <div> <div></div> <div>M1 A1=A1</div> </div> <div> <div></div> <div>M1 A1 (5)</div> </div> <p>[10]</p>
	$s$	$B$	$S$														
Mass ratios	8	19	27	anything in correct ratio													
$\bar{x}$	$\frac{3}{8} \times \frac{2}{3} r$	$\bar{x}$	$\frac{3}{8} r$														



Question Number	Scheme	Marks
Q4.	 <p> <math>\uparrow \quad T \cos \theta = 40</math>  <math>\rightarrow \quad T \sin \theta = 30</math>  leading to <math>T = 50</math> </p> <p> <math>E = \frac{\lambda x^2}{2a} = 10</math>  HL <math>T = \frac{\lambda x}{a} = 50</math> </p> <p> leading to <math>x = 0.4</math>  <math>OP = 0.5 + 0.4 = 0.9 \text{ (m)}</math> </p>	<p>M1 attempt at both equations</p> <p>M1 A1 A1 M1 A1</p> <p>B1</p> <p>M1</p> <p>M1 A1</p> <p>A1ft (10) [10]</p>

Question Number	Scheme	Marks
Q5.	<p>(a)</p>  $\frac{1}{2}m \times 2ag - \frac{1}{2}mv^2 = mg(2a - 3a \sin \theta)$ <p>leading to <math>v^2 = 2ga(3 \sin \theta - 1)</math> *</p> <p>(b) minimum value of <math>T</math> is when <math>v = 0 \Rightarrow \sin \theta = \frac{1}{3}</math></p> $T = mg \sin \theta = \frac{mg}{3}$ <p>maximum value of <math>T</math> is when <math>\theta = \frac{\pi}{2} \quad (v^2 = 4ag)</math></p> $\uparrow \quad T = \frac{mv^2}{3a} + mg$ $= \frac{7mg}{3}$ $\left( \frac{mg}{3} \leq T \leq \frac{7mg}{3} \right)$	<p> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <div style="border-left: 1px solid black; height: 20px; width: 10px; margin-bottom: 5px;"></div> <div style="border-left: 1px solid black; height: 20px; width: 10px;"></div> </div> <div> M1 A1=A1  M1 A1 (5) </div> </div></p> <p>cs0</p> <p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>A1 (6)</p> <p>[11]</p>

Question Number	Scheme	Marks
Q6.	<p>(a)</p>  <p> <math>\uparrow \quad R = mg</math>              Use of limiting friction, <math>F_r = \mu R</math>  <math>\leftarrow \quad \mu R = \frac{m28^2}{120}</math>  <math>\mu = \frac{28^2}{120 \times 9.8} = \frac{2}{3} \quad *</math> </p> <p>(b)</p>  <p> <math>\uparrow \quad R \cos \alpha - \mu R \sin \alpha = mg</math>  <math>\leftarrow \quad \mu R \cos \alpha + R \sin \alpha = \frac{mv^2}{r}</math>  <math>\frac{\mu \cos \alpha + \sin \alpha}{\cos \alpha - \mu \sin \alpha} = \frac{v^2}{rg}</math>  <math>\frac{2 \cos \alpha + 3 \sin \alpha}{3 \cos \alpha - 2 \sin \alpha} = \frac{25}{24}</math>              leading to <math>\tan \alpha = \frac{27}{122}</math> </p> <p>Eliminating R</p> <p>Substituting values</p> <p>awrt 0.22</p>	<p>B1</p> <p>B1</p> <p>M1 A1</p> <p>cao M1 A1 (6)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>M1 A1 (8)</p> <p>[14]</p>

Question Number	Scheme	Marks
Q7.	<p>(a)</p> $\frac{1}{2}mv^2 + \frac{3mgx^2}{4a} = mg(a+x)$ <p>leading to <math>v^2 = 2g(a+x) - \frac{3gx^2}{2a}</math> *</p> <p>(b) Greatest speed is when the acceleration is zero</p> $T = \frac{\lambda x}{a} = \frac{3mgx}{2a} = mg \Rightarrow x = \frac{2a}{3}$ $v^2 = 2g\left(a + \frac{2a}{3}\right) - \frac{3g}{2a} \times \left(\frac{2a}{3}\right)^2 \left(= \frac{8ag}{3}\right)$ $v = \frac{2}{3}\sqrt{(6ag)} \quad \text{accept exact equivalents}$ <p>(c) <math>v = 0 \Rightarrow 2g(a+x) - \frac{3gx^2}{2a} = 0</math></p> $3x^2 - 4ax - 4a^2 = (x-2a)(3x+2a) = 0$ $x = 2a$ <p>At D, <math>m\ddot{x} = mg - \frac{\lambda \times 2a}{a}</math></p> $ \ddot{x}  = 2g$	<p>M1 A2 (1, 0)</p> <p>cs0 A1 (4)</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1ft</p> <p>A1 (6)</p> <p>[14]</p>
	<p>Alternative to (b)</p> $v^2 = 2g(a+x) - \frac{3gx^2}{2a}$ <p>Differentiating with respect to <math>x</math></p> $2v \frac{dv}{dx} = 2g - \frac{3gx}{a}$ $\frac{dv}{dx} = 0 \Rightarrow x = \frac{2a}{3}$ $v^2 = 2g\left(a + \frac{2a}{3}\right) - \frac{3g}{2a} \times \left(\frac{2a}{3}\right)^2 \left(= \frac{8ag}{3}\right)$ $v = \frac{2}{3}\sqrt{(6ag)} \quad \text{accept exact equivalents}$	<p>M1 A1</p> <p>M1</p> <p>A1 (4)</p>

Question Number	Scheme	Marks
Q7.	<p><i>Alternative approach using SHM for (b) and (c)</i></p> <p>If SHM is used mark (b) and (c) together placing the marks in the grid as shown.</p> <p>Establishment of equilibrium position</p> $T = \frac{\lambda x}{a} = \frac{3mge}{2a} = mg \Rightarrow e = \frac{2a}{3}$ <p>N2L, using y for displacement from equilibrium position</p> $m\ddot{y} = mg - \frac{\frac{3}{2}mg(y+e)}{a} = -\frac{3g}{2a}y$ $\omega^2 = \frac{3g}{2a}$ <p>Speed at end of free fall <math>u^2 = 2ga</math></p> <p>Using A for amplitude and <math>v^2 = \omega^2(a^2 - x^2)</math></p> $u^2 = 2ga \text{ when } y = -\frac{2}{3}a \Rightarrow 2ga = \frac{3g}{2a}\left(A^2 - \frac{4a^2}{9}\right)$ $A = \frac{4a}{3}$ <p>Maximum speed <math>A\omega = \frac{4a}{3} \times \sqrt{\left(\frac{3g}{2a}\right)} = \frac{2}{3}\sqrt{6ag}</math></p> <p>Maximum acceleration <math>A\omega^2 = \frac{4a}{3} \times \frac{3g}{2a} = 2g</math></p>	<p>bM1 bA1</p> <p>bM1 bA1</p> <p>cM1</p> <p>cM1</p> <p>cA1</p> <p>cM1 cA1</p> <p>cA1</p>



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# Mark Scheme (Results) Summer 2010

GCE

GCE Mechanics M3 (6679/01)



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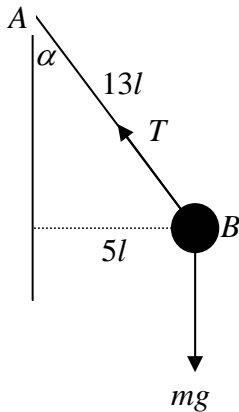
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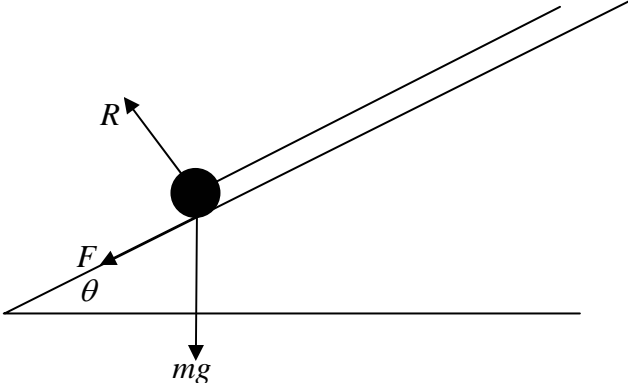
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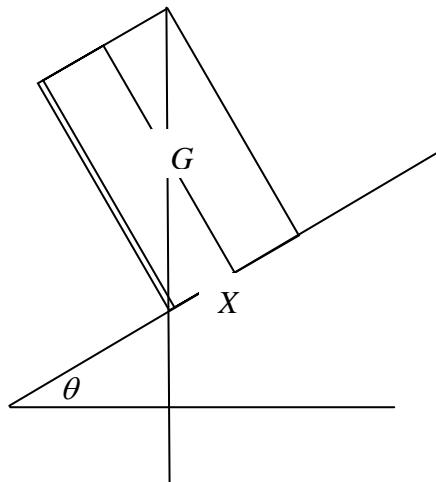
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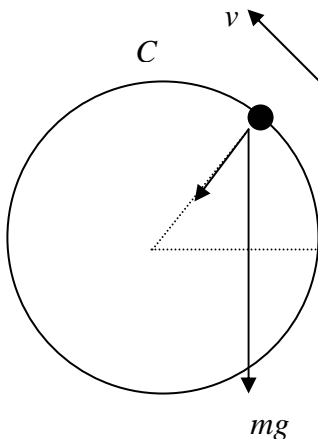
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Mechanics M3 6679  
Mark Scheme

Question Number	Scheme	Marks
Q1	 <p>(a)</p> $\cos \alpha = \frac{12}{13}$ $\text{R}(\uparrow) \quad T \cos \alpha = mg$ $T \times \frac{12}{13} = mg$ $T = \frac{13}{12} mg \quad \text{oe}$	<p>B1</p> <p>M1</p> <p>A1 (3)</p>
(b)	<p>Eqn of motion <math>T \sin \alpha = m \frac{v^2}{5l}</math></p> $\frac{13mg}{12} \times \frac{5}{13} = m \frac{v^2}{5l}$ $v^2 = \frac{25gl}{12}$ $v = \frac{5}{2} \sqrt{\frac{gl}{3}} \quad \left( \text{accept } 5\sqrt{\frac{gl}{12}} \text{ or } \sqrt{\frac{25gl}{12}} \text{ or any other equiv} \right)$	<p>M1 A1</p> <p>M1 dep</p> <p>A1 (4)</p> <p>[7]</p>

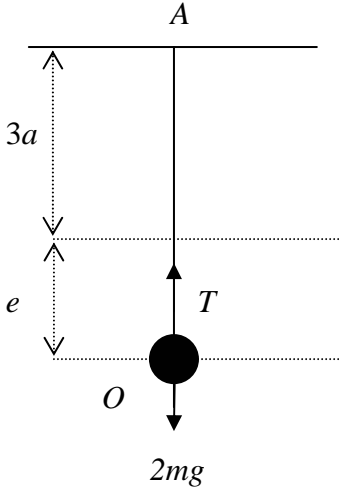
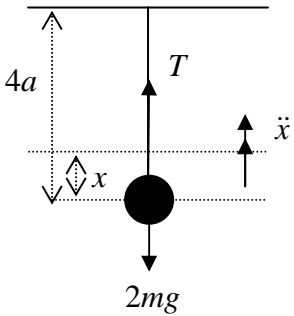
Question Number	Scheme	Marks
Q2 (a)	$F = (-) \frac{k}{x^2}$ $mg = (-) \frac{k}{R^2}$ $F = \frac{mgR^2}{x^2} *$	M1 M1 A1 (3)
(b)	$m\ddot{x} = -\frac{mgR^2}{x^2}$ $v \frac{dv}{dx} = -\frac{gR^2}{x^2}$ $\frac{1}{2}v^2 = \int \left( -\frac{gR^2}{x^2} \right) dx$ $\frac{1}{2}v^2 = \frac{gR^2}{x} (+c)$ $x = R, v = 3U \quad \frac{9U^2}{2} = gR + c$ $\frac{1}{2}v^2 = \frac{gR^2}{x} + \frac{9U^2}{2} - gR$ $x = 2R, v = U \quad \frac{1}{2}U^2 = \frac{gR^2}{2R} + \frac{9U^2}{2} - gR$ $U^2 = \frac{gR}{8}$ $U = \sqrt{\frac{gR}{8}}$	M1 M1 M1 dep on 1st M mark A1 M1 dep on 3rd M mark M1 dep on 3rd M mark A1 (7) [10]

Question Number	Scheme	Marks
Q3	 $\text{EPE lost} = \frac{\lambda \times 0.6^2}{2 \times 0.9} - \frac{\lambda \times 0.1^2}{2 \times 0.9} \left( = \frac{7}{36} \lambda \right)$ $R(\uparrow) \quad R = mg \cos \theta$ $= 0.5g \times \frac{4}{5} = 0.4g$ $F = \mu R = 0.15 \times 0.4g$ $\text{P.E. gained} = \text{E.P.E. lost} - \text{work done against friction}$ $0.5g \times 0.7 \sin \theta = \frac{\lambda \times 0.6^2}{2 \times 0.9} - \frac{\lambda \times 0.1^2}{2 \times 0.9} - 0.15 \times 0.4g \times 0.7$ $0.1944\lambda = 0.5 \times 9.8 \times 0.7 \times \frac{3}{5} + 0.15 \times 0.4 \times 9.8 \times 0.7$ $\lambda = 12.70 \dots$ $\lambda = 13 \text{ N} \quad \text{or } 12.7$	<p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1 A1</p> <p>A1</p> <p>[9]</p>

Question Number	Scheme				Marks
Q4 (a)					M1 A1  B1  M1 A1ft A1 (6)
		cone	container	cylinder	
	mass ratio	$\frac{4\pi l^3}{3}$	$\frac{68\pi l^3}{3}$	$24\pi l^3$	
		4	68	72	
	dist from $O$	$l$	$\bar{x}$	$3l$	
Moments: $4l + 68\bar{x} = 72 \times 3l$ $\bar{x} = \frac{212l}{68} = \frac{53}{17}l$ accept $3.12l$					
(b)	<div></div> $GX = 6l - \bar{x} \text{ seen}$ $\tan \theta = \frac{2l}{6l - \bar{x}}$ $= \frac{2 \times 17}{49}$ $\theta = 34.75... = 34.8 \text{ or } 35$				M1  M1 A1   A1 (4) [10]

Question Number	Scheme	Marks
Q5		
(a)	Energy: $mga \sin \theta = \frac{1}{2}m \times 5ag - \frac{1}{2}mv^2$ $v^2 = 5ag - 2ag \sin \theta$	M1 A1 A1 (3)
(b)	Eqn of motion along radius: $T + mg \sin \theta = \frac{mv^2}{a}$ $T = \frac{m}{a}(5ag - 2ag \sin \theta) - mg \sin \theta$ $T = mg(5 - 3 \sin \theta)$	M1 A1 M1 A1 (4)
(c)	At C, $\theta = 90^\circ$ $T = mg(5 - 3) = 2mg$ $T > 0 \therefore P$ reaches C	M1 A1 A1 (3)
(d)	Max speed at lowest point ( $\theta = 270^\circ$ ; $v^2 = 5ag - 2ag \sin 270$ ) $v^2 = 5ag + 2ag$ $v = \sqrt{7ag}$	M1 A1 (2) [12]

Question Number	Scheme	Marks
Q6 (a)	$\frac{d^2x}{dt^2} = -\frac{3}{(t+1)^2}$ $\frac{dx}{dt} = \int -3(t+1)^{-2} dt$ $= 3(t+1)^{-1} (+c)$ $t=0, v=2 \quad 2=3+c \quad c=-1$ $\frac{dx}{dt} = \frac{3}{t+1} - 1 \quad *$	M1  M1 A1  M1 A1 (5)
(b)	$x = \int \left( \frac{3}{t+1} - 1 \right) dt$ $= 3 \ln(t+1) - t \quad (+c')$ $t=0, x=0 \Rightarrow c' = 0$ $x = 3 \ln(t+1) - t$ $v=0 \Rightarrow \frac{3}{t+1} = 1$ $t=2$ $x = 3 \ln 3 - 2$ $= 1.295...$ $= 1.30 \text{ m} \quad (\text{Allow } 1.3)$	M1 A1 B1  M1 A1 M1 A1 (7) [12]

Question Number	Scheme	Marks
Q7	<div style="text-align: center;">  </div> <p>(a)</p> $R(\uparrow) \quad T = 2mg$ <p>Hooke's law: <math>T = \frac{6mge}{3a}</math></p> $2mg = \frac{6mge}{3a}$ $e = a$ $AO = 4a$	<p>B1</p> <p>M!</p> <p>A1 (3)</p>
(b)	<div style="text-align: center;">  </div> <p>H.L.</p> <p>Eqn. of motion</p> $T = \frac{6mg(a-x)}{3a} = \frac{2mg(a-x)}{a}$ $-2mg + T = 2m\ddot{x}$ $-2mg + \frac{2mg(a-x)}{a} = 2m\ddot{x}$ $-\frac{2mgx}{a} = 2m\ddot{x}$ $\ddot{x} = -\frac{g}{a}x$ <p>period <math>2\pi\sqrt{\frac{a}{g}}</math> *</p>	<p>B1ft</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1 (5)</p>



Question Number	Scheme	Marks
(c)	$v^2 = \omega^2 (a^2 - x^2)$ $v_{\max}^2 = \frac{g}{a} \left( \left( \frac{a}{4} \right)^2 - 0 \right)$ $v_{\max} = \frac{1}{4} \sqrt{ga}$	M1 A1 A1 (3)
(d)	$x = -\frac{a}{8}$ $v^2 = \frac{g}{a} \left( \frac{a^2}{16} - \frac{a^2}{64} \right)$ $= \frac{3ag}{64}$ $v^2 = u^2 + 2as$ $0 = \frac{3ag}{64} - 2gh$ $h = \frac{3a}{128}$ $\text{Total height above } O = \frac{a}{8} + \frac{3a}{128} = \frac{19a}{128}$	M1 M1 A1 A1 (4) [15]



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# Mark Scheme (Results) January 2011

GCE

## GCE Mechanics M3 (6679) Paper 1

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January 2011

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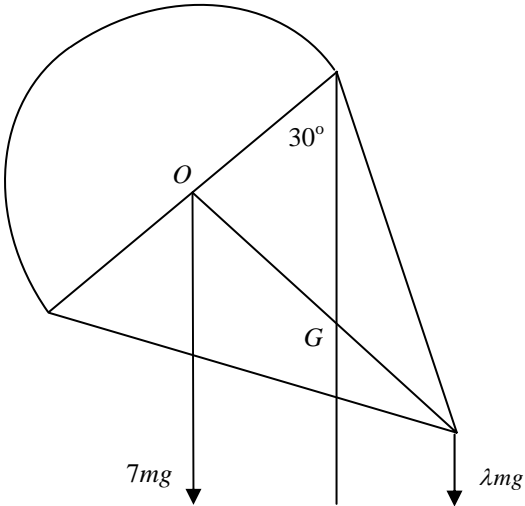
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- indep - independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper
- $\square$  The second mark is dependent on gaining the first mark

January 2011  
Mechanics M3 6679  
Mark Scheme

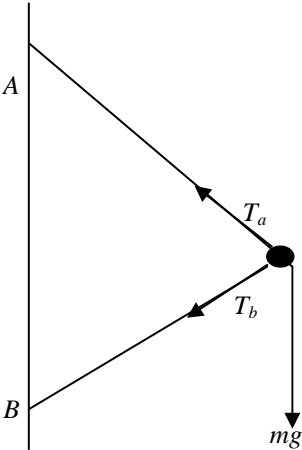
Question Number	Scheme	Marks
1.	$v \frac{dv}{dx} = 7 - 2x$ $\frac{1}{2} v^2 = 7x - x^2 \quad (+c)$ $x = 0 \quad v = 6 \Rightarrow c = 18$ $v = 0 \quad x^2 - 7x - 18 = 0$ $(x + 2)(x - 9) = 0$ $\therefore x = 9$	M1  M1A1  A1   M1 A1  [6]

Question Number	Scheme	Marks
2. (a)	<p>Mass ratio <math>\frac{4m}{\frac{3}{8}r} \quad \frac{km}{-\frac{1}{2}r} \quad \left  \begin{array}{c} (4+k)m \\ 0 \end{array} \right.</math></p> <p>Dist from <math>O</math></p> <p>Moments about <math>O</math>:  <math>\frac{3}{8}r \times 4m = \frac{1}{2}r \times km</math></p> <p><math>k = 3</math></p>	<p>B1 B1</p> <p>M1</p> <p>A1</p> <p>(4)</p>
(b)	 <p> <math>\tan 30 = \frac{OG}{r}</math>  <math>OG = \frac{\lambda}{(7+\lambda)} \times 2r</math>  <math>\frac{1}{\sqrt{3}} = \frac{\lambda}{(7+\lambda)} \times 2r \times \frac{1}{r}</math>  <math>7 + \lambda = 2\sqrt{3}\lambda</math>  <math>\lambda = \frac{7}{(2\sqrt{3}-1)} \text{ (o.e.) or } 2.84</math> </p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(4) [8]</p>



Question Number	Scheme	Marks
3.		
(a)	$\text{Vol} = \pi \int_1^2 y^2 dx = \pi \int_1^2 e^{2x} dx$ $= \frac{1}{2} \pi [e^{2x}]_1^2$ $= \frac{1}{2} \pi [e^4 - e^2]$	M1 M1 A1 A1 (4)
(b)	$\text{C of M} = \frac{\int_1^2 \pi y^2 x dx}{\text{vol}}$ $\int_1^2 e^{2x} x dx = \left[ \frac{1}{2} x e^{2x} \right]_1^2 - \int_1^2 \frac{1}{2} e^{2x} dx$ $= \left[ \frac{1}{2} x e^{2x} \right]_1^2 - \left[ \frac{1}{4} e^{2x} \right]_1^2$ $= \frac{1}{2} \times 2e^4 - \frac{1}{2} \times 1e^2 - \left( \frac{1}{4} e^4 - \frac{1}{4} e^2 \right)$ $= \left( \frac{3}{4} e^4 - \frac{1}{4} e^2 \right)$ $\text{C of M} = \frac{\pi \left( \frac{3}{4} e^4 - \frac{1}{4} e^2 \right)}{\frac{1}{2} \pi (e^4 - e^2)} = 1.656...$ $= 1.66$ (3 sf)	M1 A1 M1 A1 M1 A1 (6) [10]

Question Number	Scheme	Marks
4.		
(a)	$x = 5 \sin\left(\frac{\pi t}{3}\right)$ $\dot{x} = 5 \times \frac{\pi}{3} \cos\left(\frac{\pi t}{3}\right)$ $\ddot{x} = -5 \times \left(\frac{\pi}{3}\right)^2 \sin\left(\frac{\pi t}{3}\right)$ $\ddot{x} = -\frac{\pi^2}{9} x \quad (\because \text{S.H.M.})$	M1A1 A1 (3)
(b)	period = $\frac{2\pi}{\frac{\pi}{3}} = 6$ amplitude = 5	B1 B1 (2)
(c)	$\dots = 5 \times \frac{\pi}{3} \cos\left(\frac{\pi t}{3}\right) \quad \text{or} \quad  v_{\max}  = a\omega$ $\max. v = \frac{5\pi}{3}$	M1 A1 (2)
(d)	At A $x = 2 \quad 2 = 5 \sin\left(\frac{\pi t}{3}\right)$ $\sin \frac{\pi}{3} t = 0.4$ $t_A = \frac{3}{\pi} \times \sin^{-1} 0.4$ At B $x = 3 \quad t_B = \frac{3}{\pi} \times \sin^{-1} 0.6$ time $A \rightarrow B = \frac{3}{\pi} \times \sin^{-1} 0.6 - \frac{3}{\pi} \times \sin^{-1} 0.4$ $= 0.2215\dots = 0.22 \text{ s}$ accept awrt 0.22	M1 A1 A1 A1 (4) [11]

Question Number	Scheme	Marks
5.	 <p>(a)</p> $r = \frac{l}{\sqrt{2}}$ $\text{R}(\uparrow) \quad T_a \cos 45 = T_b \cos 45 + mg$ $T_a - T_b = mg \sqrt{2} \quad (1)$ $\text{R}(\rightarrow) \quad T_a \cos 45 + T_b \cos 45 = mr\omega^2$ $T_a \times \frac{1}{\sqrt{2}} + T_b \times \frac{1}{\sqrt{2}} = m \frac{l}{\sqrt{2}} \omega^2$ $T_a + T_b = ml\omega^2 \quad (2)$ $T_a - T_b = mg \sqrt{2} \quad (1)$ $2T_a = m(l\omega^2 + g \sqrt{2})$ $T_a = \frac{1}{2}m(l\omega^2 + g \sqrt{2})$ $T_b = ml\omega^2 - T_a$ $= \frac{1}{2}m(l\omega^2 - g \sqrt{2})$ <p>(8)</p>	<p>B1</p> <p>M1A1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>A1</p>
(b)	$T_b > 0 \quad \frac{1}{2}m(l\omega^2 - g \sqrt{2}) > 0$ $\omega^2 > \frac{g \sqrt{2}}{l} \quad *$	<p>M1</p> <p>A1</p> <p>(2)</p> <p>[10]</p>

Question Number	Scheme	Marks
6. (a)	<div data-bbox="470 253 1093 616" data-label="Diagram"> </div> <p>length <math>AP = \text{length } BP = \frac{5}{4}l</math></p> $T_a = T_b = \frac{kmg \left(\frac{1}{4}l\right)}{l} = \frac{1}{4}kmg \quad (\text{or } T = \dots)$ $R(\uparrow) \quad T_a \cos \theta + T_b \cos \theta = 3mg \quad (\text{or } 2T \cos \theta = 3mg)$ $\frac{1}{4}kmg \times \frac{3}{5} + \frac{1}{4}kmg \times \frac{3}{5} = 3mg \quad \left( \text{or } \frac{1}{2}kmg \times \frac{3}{5} = 3mg \right)$ $\frac{3}{10}kmg = 3mg$ $k = 10 \quad *$	B1 M1A1 M1A1 A1 A1 (7)
(b)	<div data-bbox="454 1149 1109 1400" data-label="Diagram"> </div> <p>initial extn <math>= \frac{13}{5}l - l = \frac{8}{5}l</math></p> $\text{E.P.E. lost} = 2 \times \frac{\lambda x^2}{2l} = 2 \times \frac{10mg}{2l} \left(\frac{8l}{5}\right)^2 = \frac{128mgl}{5}$ $\text{P.E. gained} = 3mg \times \frac{12l}{5} = \frac{36mgl}{5}$ $\frac{1}{2} \times 3mv^2 + \frac{36mgl}{5} = \frac{128mgl}{5}$ $v^2 = \frac{256gl}{15} - \frac{72gl}{15}$ $v = \sqrt{\left(\frac{184}{15}gl\right)}$	B1 M1A1 M1A1 A1 (6) [13]

Question Number	Scheme	Marks
7.	<div data-bbox="608 248 959 633" data-label="Image"> </div> <p data-bbox="169 651 204 685">(a)</p> $mgl(\cos \alpha - \cos \theta) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ $v^2 = u^2 + 2gl(\cos \alpha - \cos \theta) \quad *$	<p data-bbox="1366 651 1485 685">M1A1=A1</p> <p data-bbox="1366 752 1401 786">A1</p> <p data-bbox="1481 819 1517 853">(4)</p>
(b)	$\cos \alpha = \frac{3}{5} \quad v^2 = 2gl\left(\frac{3}{5} - \cos \theta\right) + u^2$ <p data-bbox="236 1003 738 1070">At top <math>\theta = 360^\circ \quad v^2 = 2gl\left(\frac{3}{5} - 1\right) + u^2</math></p> $v^2 > 0 \quad -2gl \times \frac{2}{5} + u^2 > 0$ $u^2 > \frac{4gl}{5}$ $u > 2\sqrt{\frac{gl}{5}} \quad *$	<p data-bbox="1366 987 1437 1021">M1A1</p> <p data-bbox="1366 1077 1406 1111">M1</p> <p data-bbox="1366 1245 1401 1279">A1</p> <p data-bbox="1481 1335 1517 1368">(4)</p>

Question Number	Scheme	Marks
(c)	<p>Equation of motion along radius at lowest point:</p> $T_1 - mg = \frac{mv^2}{l}$ $\theta = 180 \quad v^2 = 2gl\left(\frac{3}{5} + 1\right) + u^2$ $v^2 = \frac{16gl}{5} + u^2$ $T_1 = \frac{m}{l}\left(\frac{16gl}{5} + u^2\right) + mg$ $= \frac{21mg}{5} + \frac{mu^2}{l}$ <p>At highest point:</p> $T_2 + mg = \frac{mv^2}{l}$ $\theta = 360 \quad T_2 = 2mg\left(-\frac{2}{5}\right) + \frac{mu^2}{l} - mg$ $T_2 = \frac{mu^2}{l} - \frac{9mg}{5}$ $T_1 = 5T_2$ $\frac{21mg}{5} + \frac{mu^2}{l} = 5\left(\frac{mu^2}{l} - \frac{9mg}{5}\right)$ $\frac{66mg}{5} = \frac{4mu^2}{l}$ $u^2 = \frac{33gl}{10} \quad *$	<p>M1A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>(9) [17]</p>

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June 2011

GCE Mechanics M3 (6679) Paper 1



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June 2011

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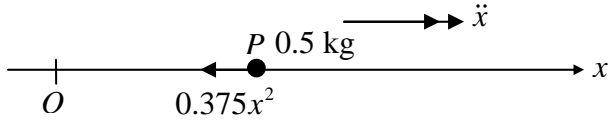
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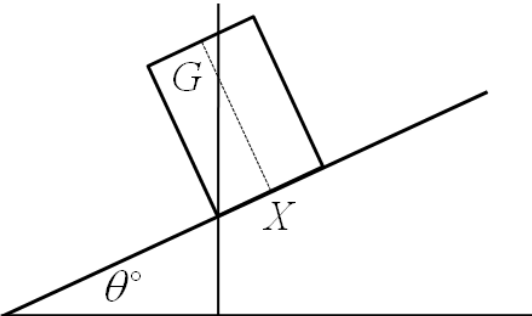
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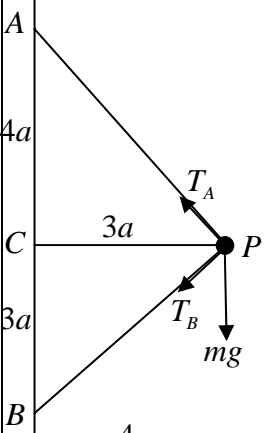
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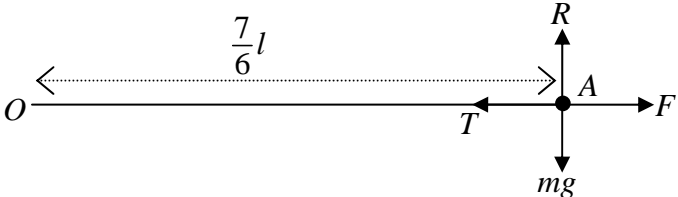
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Mark Scheme

Question Number	Scheme	Marks
1. (a)	 $0.5v \frac{dv}{dx} = -0.375x^2$ $\frac{1}{2}v^2 = -0.25x^3 + c$ $t = 0, v = 2, x = 8$ $\frac{1}{2} \times 2^2 = -0.25 \times 8^3 + c$ $c = 130$ $\therefore v^2 = -\frac{1}{2}x^3 + 260 \quad *$	<p>M1</p> <p>M1 A1</p> <p>A1</p> <p>(4)</p>
(b)	$v = 5 \quad x^3 = 520 - 50$ $x = 7.77$	<p>M1</p> <p>A1</p> <p>(2)</p> <p><b>6</b></p>

Question Number	Scheme	Marks
2.	$V = \pi \int_0^3 (9 - x^2)^2 dx = \pi \int_0^3 (81 - 18x^2 + x^4) dx$ $= \pi \left[ 81x - 6x^3 + \frac{x^5}{5} \right]_0^3 = \frac{648}{5} \pi$ $\int_0^3 \pi (9 - x^2)^2 x dx$ $= \frac{\pi}{6} \left[ -(9 - x^2)^3 \right]_0^3$ $= \frac{\pi}{6} \left[ 0 + (9)^3 \right]$ $= \pi \left[ \frac{81}{2} \times 3^2 - \frac{9}{2} \times 3^4 + \frac{1}{6} \times 3^6 \right]$ $= \frac{243}{2} \pi$	<p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p>
	$\bar{x} = \frac{\frac{243}{2}}{\frac{648}{5}} = \frac{15}{16} \quad (\text{accept } 0.94)$	<p>M1 A1</p> <p>(9) 9</p>
3. (a)	<p>Mass ratio <math>\pi(3l)^2 \times 5l\rho \quad \frac{2}{3} \pi(3l)^3 \times 2\rho \quad \left  \quad \right  \quad 81\pi l^3 \rho</math></p> <p><math>5 \quad 4 \quad 9</math></p> <p>Dist. from <math>O \quad \frac{5}{2}l \quad -\frac{3}{8} \times 3l \quad \left  \quad \right  \quad \bar{x}</math></p> <p>Moments equation:</p> $5 \times \frac{5}{2}l - 4 \times \frac{9}{8}l = 9\bar{x}$ $\bar{x} = \frac{8}{9}l$	<p>B1</p> <p>B1</p> <p>M1 A1 ft</p> <p>A1</p> <p>(5)</p>

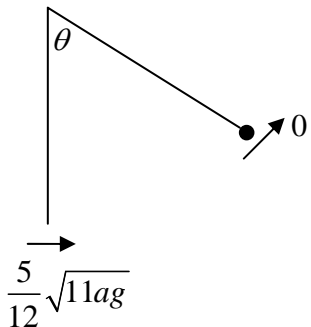
Question Number	Scheme	Marks
(b)	 $GX = 5l - \frac{8}{9}l = \frac{37}{9}l$ $\tan \theta = \frac{3l}{\frac{37}{9}l} = \frac{27}{37}$ $\theta = 36.1^\circ \text{ accept } 36^\circ, 0.63 \text{ or } 0.630 \text{ rad or better}$	<p>B1ft</p> <p>M1 A1 ft</p> <p>A1</p> <p>(4) 9</p>

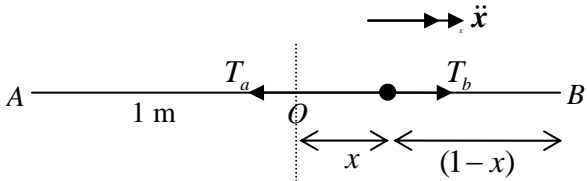
Question Number	Scheme	Marks
4. (a)	 <p> <math>\cos \theta = \frac{4}{5}</math> or <math>\sin \theta = \frac{3}{5}</math>  R (vert) <math>T_B \cos 45 + mg = T_A \cos \theta</math>  <math>\frac{1}{\sqrt{2}} T_B + mg = \frac{4}{5} T_A</math>  R (horiz) <math>T_A \sin \theta + T_B \cos 45 = m \times 3a\omega^2</math>  <math>\frac{3}{5} T_A + \frac{1}{\sqrt{2}} T_B = 3ma\omega^2</math>  <math>\frac{3}{5} T_A - mg = 3ma\omega^2 - \frac{4}{5} T_A</math>  <math>\frac{7}{5} T_A = 3ma\omega^2 + mg</math>  <math>T_A = \frac{5}{7} m(3a\omega^2 + g)</math> * </p>	<p>B1</p> <p>M1 A1</p> <p>M1 A1=A1</p> <p>M1</p> <p>A1 (8)</p>
(b)	<p> <math>T_b = \sqrt{2} \left( \frac{4}{5} T_a - mg \right)</math>  <math>= \sqrt{2} \left( \frac{4}{7} m(3a\omega^2 + g) - mg \right)</math>  <math>= \frac{3\sqrt{2}}{7} m(4a\omega^2 - g)</math> oe </p>	<p>M1</p> <p>A1 (2)</p>

Question Number	Scheme	Marks
(c)	$T_b \geq 0 \Rightarrow 4a\omega^2 \geq g$ $\omega^2 \geq \frac{g}{4a}$ $\omega \geq \frac{1}{2}\sqrt{\frac{g}{a}} \quad *$ <p>(Allow strict inequalities in (c).)</p>	<p>M1</p> <p>A1</p> <p>(2)</p> <p><b>12</b></p>
5. (a)	 $T = \frac{3mg}{l} \left( \frac{1}{6}l \right) = \frac{1}{2}mg$ $R(\uparrow) \quad R = mg \quad R(\rightarrow) \quad F = T = \frac{1}{2}mg$ $F \leq \mu R$ $\frac{1}{2}mg \leq \mu mg$ $\mu \geq \frac{1}{2} \quad *$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(4)</p>
(b)	$\text{E.P.E. lost} = \frac{1}{2} \times \frac{3mg}{l} \left( \frac{1}{2}l \right)^2 = \frac{3mgl}{8}$ $\text{Work done by friction} = \frac{1}{2}mg \left( \frac{l}{2} \right)$ $\frac{3mgl}{8} = \frac{1}{2}mv^2 + \frac{1}{2}mg \left( \frac{l}{2} \right)$ $v^2 = \frac{gl}{4}$ $v = \frac{1}{2}\sqrt{gl}$	<p>B1</p> <p>B1</p> <p>M1 A1ft</p> <p>A1</p> <p>(5)</p>

[illegible]



Question Number	Scheme	Marks
(b)	<p>Energy to C:</p> $\frac{1}{2} \times mU^2 - \frac{1}{2} m(3\sqrt{ag})^2 = mag$ $U^2 = 2ag + 9ag$ $U = \sqrt{11ga}$	<p>M1</p> <p>A1</p> <p>(2)</p>
(c)	 <p>Energy from C to rest:</p> $\frac{1}{2} \times m \times \left( \frac{5}{12} \sqrt{11ag} \right)^2 = mga(1 - \cos \theta)$ $\frac{25}{144} \times 11ag = 2ga(1 - \cos \theta)$ $\cos \theta = \frac{1}{2} \left( 2 - \frac{25 \times 11}{144} \right)$ $\theta = 87.4 \dots$ $\theta = 87^\circ \text{ (or 1.5 rad) or better}$	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p>(4) 12</p>

Question Number	Scheme	Marks
7.	 <p>(a) Total extn. = 0.6  <math>T_b = \frac{\lambda \times \text{ext}}{l} = \frac{2(0.3-x)}{0.7} = \frac{2}{7}(3-10x) \quad *</math></p>	M1 A1 (2)
(b)	$T_a = \frac{2(x+0.3)}{0.7} \quad \left( = \frac{2}{7}(10x+3) \right)$	B1 (1)
(c)	$T_b - T_a = 0.5\ddot{x}$ $\frac{2}{7}(3-10x) - \frac{2}{7}(10x+3) = 0.5\ddot{x}$ $2 \times \left( -\frac{20x}{7} \right) = 0.5\ddot{x}$ $\ddot{x} = -\frac{40}{7 \times 0.5} x$ $(\therefore \text{S.H.M.})$ $\text{Period} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{7 \times 0.5}{40}} = 2\pi \sqrt{\frac{7}{80}} \quad *$	M1 A1 ft M1 A1 M1 A1 (6)
(d)	$v_{\max} = a\omega = 0.2\sqrt{\frac{80}{7}} \text{ o.e. or a.w.r.t. } 0.68 \text{ m s}^{-1}$	M1 A1 (2)
(e)	$x = a \cos \omega t = 0.2 \cos \left( \sqrt{\frac{80}{7}} t \right)$ $x = -0.1 \quad -\frac{0.1}{0.2} = \cos \left( \sqrt{\frac{80}{7}} t \right)$ $t = \sqrt{\frac{7}{80}} \cos^{-1}(-0.5)$ $t = \sqrt{\frac{7}{80}} \times \frac{2\pi}{3} = \frac{\pi}{3} \sqrt{\frac{7}{20}} \text{ o.e. (accept a.w.r.t. } 0.62) \text{ s}$	M1 A1 M1 A1 (4)

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Mark Scheme (Results)

January 2012

GCE Mechanics M3 (6679) Paper 1

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January 2012

Publications Code UA030776

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  - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

## General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

### Method mark for solving 3 term quadratic:

#### 1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$ , where  $|pq| = |c|$ , leading to  $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = \dots$

#### 2. Formula

Attempt to use correct formula (with values for  $a$ ,  $b$  and  $c$ ), leading to  $x = \dots$

#### 3. Completing the square

Solving  $x^2 + bx + c = 0$  :  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$ ,  $q \neq 0$ , leading to  $x = \dots$

### Method marks for differentiation and integration:

#### 1. Differentiation

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

#### 2. Integration

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

### Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.



**January 2012**  
**6679 Mechanics M3**  
**Mark Scheme**

Question Number	Scheme	Marks
<p style="text-align: center;"><b>1.</b></p>	$\text{EPE} = \frac{\lambda \times 0.5^2}{1.2}$ <p>GPE lost = EPE gained</p> $0.8 \times 9.8 \times 1.1 = \frac{\lambda \times 0.5^2}{1.2}$ <p><math>\lambda = 41.4 \text{ N}</math> or <math>41 \text{ N}</math></p>	<p>B1</p> <p>M1 (used)</p> <p>A1ft</p> <p>A1</p> <p style="text-align: right;"><b>4</b></p>

Question Number	Scheme	Marks
2.		
(a)	$T = \frac{2\pi}{\omega} = \frac{2\pi}{3}, \quad \omega = 3$ $ a  = \omega^2 x = 9 \times 0.2 = 1.8 \text{ ms}^{-2} \text{ towards } C$	B1 M1 A1 (3)
(b)	$v^2 = \omega^2 (a^2 - x^2) = 9(0.25 - 0.04) = 1.89$ $v = 1.37 \text{ ms}^{-1}$	M1 A1 (2)
(c)	$x = 0.5 \sin 3t = 0.2$ $t = \frac{1}{3} \sin^{-1} 0.4 \approx 0.137 \text{ s}$	M1 A1ft A1 (3) <b>8</b>

Question Number	Scheme	Marks
3.		
(a)	$a = v \frac{dv}{dx} = \frac{10}{x+6} \times \frac{-10}{(x+6)^2}, = \frac{-100}{(x+6)^3}$ $= \frac{-100}{(14+6)^3} = -\frac{1}{80} \text{ms}^{-2}$	M1 M1, A1  A1  (4)
(b)	$\frac{dx}{dt} = \frac{10}{x+6} \Rightarrow \int x+6 dx = \int 10 dt$ $\left[ \frac{x^2}{2} + 6x \right]_2^{14} = [10t]_1^T$ $\frac{196}{2} + 6 \times 14 - 2 - 12 = 10T - 10$ $178 = T \quad T = 17.8(\text{s})$	M1 M1 M1 A1 M1 A1  (6) <b>10</b>

Question Number	Scheme	Marks
<p><b>4.</b></p> <p><b>(a)</b></p>	<div data-bbox="435 405 1054 752" data-label="Diagram"> </div> <p> <math>\uparrow T \cos 60^\circ = 0.5g, T = g \quad (1)</math>            Extension in the string = <math>x, T = \frac{\lambda x}{a} = \frac{19.6x}{0.8}</math>            Using (1), <math>g = 24.5x, x = 0.4 \text{ m}^*</math> </p> <p><b>(b)</b></p> <p> <math>\rightarrow T \sin 60^\circ = 0.5 \times r \times \omega^2 \quad (2)</math>            Using (2) <math>g \sin 60^\circ = 0.5 \times (0.8 + 0.4) \sin 60^\circ \omega^2</math>  <math>\omega^2 = \frac{2g}{1.2}, \omega = \sqrt{\frac{5g}{3}} \quad (4.04 \text{ or } 4.0)</math> </p>	<p>M1, A1</p> <p>B1</p> <p>M1, A1</p> <p>(5)</p> <p>M1 A1</p> <p>M1 A1</p> <p>A1</p> <p>(5)</p> <p><b>10</b></p>

Question Number	Scheme	Marks
5. (a)	<p>Distance of P from the centre of the Earth = <math>R + x</math></p> $F = \frac{k}{(R + x)^2}$ <p><math>x = 0, F = mg, \quad k = mg(R)^2</math></p> $F = \frac{mgR^2}{(R + x)^2} \quad *$	<p>M1 A1</p> <p>A1</p> <p>(3)</p>
(b)	<p><math>F = ma, \quad -\frac{gR^2}{(R + x)^2} = v \frac{dv}{dx}</math></p> $\int_v^{\sqrt{\frac{gR}{2}}} v dv = \int_R^{2R} -\frac{gR^2}{(R + x)^2} dx$ $\left[ \frac{1}{2} v^2 \right]_V^{\sqrt{\frac{gR}{2}}} = \left[ \frac{gR^2}{R + x} \right]_R^{2R}$ $\frac{1}{2} \times \frac{gR}{2} - \frac{1}{2} V^2 = \frac{gR^2}{3R} - \frac{gR^2}{2R} = -\frac{gR}{6}$ $\frac{V^2}{2} = \frac{gR}{4} + \frac{gR}{6} = \frac{5gR}{12} \quad V^2 = \frac{5gR}{6}, \quad V = \sqrt{\frac{5gR}{6}}$	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1, A1</p> <p>(9)</p> <p><b>12</b></p>

Question Number	Scheme	Marks
6. (a)	<p>GPE gained = <math>mgl(1 - \cos \theta)</math></p> <p>Conservation of energy: <math>\frac{1}{2}m\frac{11gl}{4} = mgl(1 - \cos \theta) + \frac{1}{2}mv^2</math></p> $v^2 = gl\left(\frac{11}{4} - 2 + 2\cos \theta\right) = gl\left(\frac{3}{4} + 2\cos \theta\right)$ <p>Resolving towards the centre of the circle:</p> $T - mg \cos \theta = \frac{mv^2}{l}$ $T - mg \cos \theta = mg\left(\frac{3}{4} + 2\cos \theta\right)$ $T = mg\left(\frac{3}{4} + 3\cos \theta\right) = 3mg\left(\cos \theta + \frac{1}{4}\right) \quad *$ <p>(b) <math>T = 0 \Rightarrow \cos \theta = -\frac{1}{4}</math></p> $v^2 = gl\left(\frac{3}{4} + 2\cos \theta\right) = \frac{gl}{4}, \quad v = \sqrt{\frac{gl}{4}}$ <p>(c) Horizontal component of velocity at B</p> $= \sqrt{\frac{gl}{4}} \times \cos(180 - \theta) = \frac{1}{4}\sqrt{\frac{gl}{4}}$ <p>Extra height <math>h \Rightarrow mgh + \frac{1}{2}m\frac{gl}{64} = \frac{1}{2}m\frac{gl}{4}</math></p> $h = \left(\frac{1}{8} - \frac{1}{128}\right)l = \frac{15l}{128} \quad (0.117l)$ <p>OR: Using <math>h = \frac{v^2 \sin^2 \theta}{2g} = \frac{\frac{gl}{4} \times \frac{15}{16}}{2g} = \frac{15l}{128}</math></p> <p>OR: Using <math>v^2 = u^2 + 2as</math>, <math>0 = \frac{15gl}{64} - 2gh</math>, <math>h = \frac{15l}{128}</math></p>	<p>M1A1 A1</p> <p>M1</p> <p>A1 A1</p> <p>M1</p> <p>A1</p> <p>(8)</p> <p>M1</p> <p>M1, A1</p> <p>(3)</p> <p>B1ft</p> <p>M1 A1</p> <p>A1</p> <p>(4)</p> <p>15</p>

Question Number	Scheme	Marks
7.		
(a)	$\int \pi y^2 dx = \frac{\pi}{4} \int x^2 (6-x)^2 dx = \frac{\pi}{4} \int 36x^2 - 12x^3 + x^4 dx$ $= \frac{\pi}{4} \left[ 12x^3 - 3x^4 + \frac{x^5}{5} \right]_2^6 = \frac{\pi}{4} \times \frac{1024}{5}$ <p style="text-align: right;">(160.8....)</p> $\int \pi y^2 x dx = \frac{\pi}{4} \int x^3 (6-x)^2 dx = \frac{\pi}{4} \int 36x^3 - 12x^4 + x^5 dx$ $= \frac{\pi}{4} \left[ 9x^4 - \frac{12}{5}x^5 + \frac{1}{6}x^6 \right]_2^6 = \frac{\pi}{4} \times \frac{10496}{15}$ <p style="text-align: right;">(549.5.....)</p> $\Rightarrow \bar{x} = \frac{10496}{15} \times \frac{5}{1024} = 3.416.....$ <p>Required distance <math>\approx 3.42 - 2 = 1.42(\text{cm})</math> *</p>	<p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p style="text-align: right;">(9)</p>
(b)	<p>Base has radius <math>\frac{1}{2} \times 2 \times 4 = 4 \text{ cm}</math></p> <p>About to topple <math>\Rightarrow \tan \alpha = \frac{4}{1.42}</math></p> <p style="text-align: center;"><math>\alpha \approx 70.5^\circ</math></p>	<p>B1</p> <p>M1 A1</p> <p>A1</p> <p style="text-align: right;">(4)</p>
(c)	<p>Parallel to slope: <math>F = mg \sin \beta</math></p> <p>Perpendicular to the slope: <math>R = mg \cos \beta</math></p> <p>About to slip: <math>F = \mu R</math></p> <p style="text-align: center;"><math>\tan \beta = \mu = 0.3, \quad \beta \approx 16.7^\circ</math></p>	<p>M1 A1</p> <p>A1</p> <p style="text-align: right;">(3)</p>
		<b>16</b>





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# Mark Scheme (Results)

Summer 2012

GCE Mechanics M3  
(6679) Paper 1

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Summer 2012

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**Summer 2012  
6679 Mechanics 3  
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## General Principles for Mechanics Marking

Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

Omission or extra g in a resolution is accuracy error not method error.

Omission of mass from a resolution is method error.

Omission of a length from a moments equation is a method error.

Omission of units or incorrect units is not (usually) counted as an accuracy error.

DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

Any numerical answer which comes from use of  $g = 9.8$  should be given to 2 or 3 SF.

Use of  $g = 9.81$  should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *ONCE* per complete question.

However, premature approximation should be penalised every time it occurs.

**MARKS MUST BE ENTERED IN THE SAME ORDER AS THEY APPEAR ON THE MARK SCHEME.**

In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.

Accept column vectors in all cases.

**Summer 2012**  
**6679 Mechanics M3**  
**Mark Scheme**

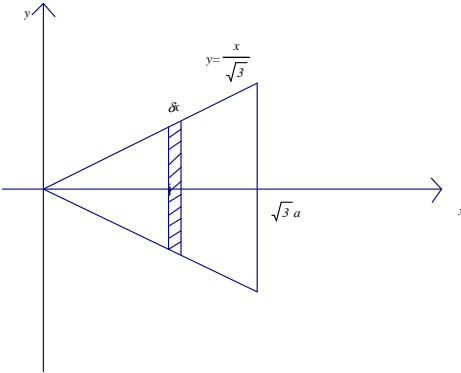
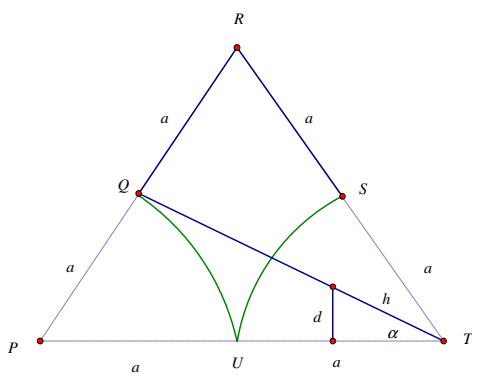
Question Number	Scheme	Marks
1(a)	Use of $a = v \frac{dv}{dx}$ or $a = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$ $a = 2e^{-x} - 2e^{-x}$ or $v^2 = 4e^{-2x}$ $a = -4e^{-2x}$	M1 A1 A1 (3)
(b)	Separate the variables and attempt to integrate: $\int 2dt = \int e^x dx$ $2t = e^x + C$ $t=0, x=0 \Rightarrow C=-1, 2t = e^x - 1$ $x = \ln(2t + 1)$	M1 A1A1 M1A1 A1 (6) <b>9</b>
2(a)	$T = \frac{2\pi}{\omega} \Rightarrow \omega = 4$ Use of $v^2 = \omega^2 (v^2 - x^2)$ , or $v = a\omega$ $a = 1.5$ (m)	B1 M1 A1 (3)
(b)	Use of max. accn. = $\omega^2 a$ $24 \text{ ms}^{-2}$	M1 A1 (2)
(c)	$x = a \sin \omega t$ with their values for $a$ & $\omega$ $1 = 1.5 \sin 4t$ (with their 1.5 & 4) and attempt to solve for $t$ $t = 0.18$ (or awrt)	B1 M1 A1 (3) <b>8</b>

Question Number	Scheme	Marks
3	<div data-bbox="300 331 794 833" data-label="Diagram"> </div> <p data-bbox="284 891 513 967"> <math>\cos \theta = \frac{0.2}{0.6} \left( = \frac{1}{3} \right)</math> </p> <p data-bbox="284 981 523 1012">Resolve vertically:</p> <p data-bbox="284 1016 855 1057"> <math>T_A \cos \theta = T_B \cos \theta + mg \quad (T_A = T_B + 3mg)</math> </p> <p data-bbox="284 1061 699 1093">Acceleration towards the centre:</p> <p data-bbox="284 1102 1168 1182"> <math>T_A \sin \theta + T_B \sin \theta = m \times 0.6 \sin \theta \times \omega^2 \quad \left( T_A + T_B = 5 \times \frac{3}{5} \times 100 = 300 \right)</math> </p> <p data-bbox="284 1191 1152 1223">Substitute values for <math>\omega</math> and trig functions and solve to find <math>T_A</math> or <math>T_B</math></p> <p data-bbox="284 1227 868 1258"> <math>T_B + 147 + T_B = 300, \quad 2T_B = 300 - 147 = 153</math> </p> <p data-bbox="284 1272 721 1303"> <math>T_A = 223.5(\text{N}) \quad , \quad T_B = 76.5(\text{N})</math> </p> <p data-bbox="284 1317 635 1348"> <math>T_A = 224 \text{ or } 220 \quad T_B = 76</math> </p> <p data-bbox="284 1361 651 1393"> <math>T_B = 76.5 \text{ or } 77 \quad T_A = 223</math> </p>	<p data-bbox="1286 891 1321 922">B1</p> <p data-bbox="1286 981 1327 1012">M1</p> <p data-bbox="1286 1016 1375 1048">A2,1,0</p> <p data-bbox="1286 1061 1327 1093">M1</p> <p data-bbox="1286 1102 1375 1133">A2,1,0</p> <p data-bbox="1286 1191 1327 1223">M1</p> <p data-bbox="1286 1227 1375 1258">A1,A1</p> <p data-bbox="1426 1415 1487 1482" style="text-align: right;"> (10)  <b>10</b> </p>



Question Number	Scheme				Marks
4 (a)		volume	Mass ratio	C of M from V	B1, B1    M1A1  A1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                <

Question Number	Scheme	Marks
5(a)	<div data-bbox="501 322 1027 604" data-label="Image"> </div> <p>Conservation of energy : Loss in GPE = gain in KE</p> $mga(\cos \alpha - \cos \theta) = \frac{1}{2}mv^2$ <p>Substitute for <math>\cos \alpha</math> and rearrange to <b>given answer</b>:</p> $v^2 = \frac{2mga}{m} \left( \frac{3}{5} - \cos \theta \right) = \frac{2ga}{5} (3 - 5 \cos \theta) \quad *$	<p>M1 A2,1,0</p> <p>A1</p> <p>(4)</p>
(b)	<p>Considering the acceleration towards the centre of the hemisphere:</p> $mg \cos \theta - R = \frac{mv^2}{a}$ <p>Substitute for <math>v^2</math> to form expression for <math>R</math>:</p> $R = mg \cos \theta - \frac{mv^2}{a} = mg(3 \cos \theta - 2 \cos \alpha) \left( = mg \left( 3 \cos \theta - \frac{6}{5} \right) \right)$ <p>Loses contact with the surface when <math>R = 0</math></p> $\cos \theta = \frac{2}{5}$ $v^2 = \frac{2ga}{5}, \quad v = \sqrt{\frac{2ga}{5}}$	<p>M1 A2,1,0</p> <p>DM1 A1</p> <p>M1 A1</p> <p>A1</p> <p>(8) <b>12</b></p>
Alt:	$R = 0 \Rightarrow mg \cos \theta = \frac{mv^2}{a}$ $\cos \theta = \frac{v^2}{ga}$ <p>Substitute in given (a) <math>v^2 = \frac{2ga}{5} \left( 3 - 5 \frac{v^2}{ga} \right)</math></p> $v^2 = \frac{6ga}{5} - 2v^2, \quad 3v^2 = \frac{6ga}{5}$ $v = \sqrt{\frac{2ga}{5}}$	<p>DM1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>

Question Number	Scheme	Marks
6(a)	 <p>Mass of lamina = <math>\rho \frac{1}{2} \times 2a \times \sqrt{3}a = \sqrt{3}\rho a^2</math></p> <p><math>\sum \rho x \times \frac{2x}{\sqrt{3}} \times \delta x = \rho \int_0^{\sqrt{3}a} \frac{2x^2}{\sqrt{3}} dx</math></p> <p><math>= \rho \left[ \frac{2x^3}{3\sqrt{3}} \right]_0^{\sqrt{3}a}</math></p> <p><math>= \rho \frac{2 \times 3\sqrt{3}a^3}{3\sqrt{3}} = 2\rho a^3</math></p> <p>Distance from vertex = <math>\frac{2\rho a^3}{\sqrt{3}\rho a^2} = \frac{2}{3}a\sqrt{3} \quad **</math></p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1A1</p>
(b)	 <p>Area of each sector = <math>\frac{1}{6} \pi a^2</math></p> <p>Using sector formula, <math>d = h \sin \alpha = \frac{2a \sin \alpha}{3\alpha} \sin \alpha = \frac{a}{3} \times \frac{1}{2} = \frac{a}{6}</math></p> <p>Taking moments: <math>\left( \sqrt{3}a^2 - 2 \times \frac{\pi a^2}{6} \right) D = \sqrt{3}a^2 \times \frac{\sqrt{3}a}{3} - 2 \times \frac{\pi a^2}{6} \times \frac{a}{\pi}</math></p>	<p>B1</p> <p>B2,1,0</p> <p>M1A1</p>

(6)

Question Number	Scheme	Marks
	$D = \frac{\frac{2a^3}{3}}{\left(\sqrt{3} - \frac{\pi}{3}\right)a^2} = \frac{2a}{3\sqrt{3} - \pi} \quad **$	A1 (6) <b>12</b>

Question Number	Scheme	Marks
7(a)	Use of $T = \frac{\lambda x}{a} = mg$ $T = \frac{24.5x}{0.75} = 0.5g$ $x = \frac{0.75 \times 0.5g}{24.5} = 0.15, \quad AE = 0.75 + 0.15 = 0.9 \text{ (m)} \quad (**)$	M1 A1 A1 (3)
(b)	Using gain in EPE = loss in GPE $\frac{\lambda x^2}{2a} = \frac{24.5x^2}{1.5} = \dots$ $\dots = 0.5g(0.75 + x)$ Form quadratic in $x$ and attempt to solve for $x$ : $24.5x^2 = 5.5125 + 7.35x, \quad 24.5x^2 - 7.35x - 5.5125 = 0,$ $x = \frac{7.35 \pm \sqrt{7.35^2 + 4 \times 24.5 \times 5.5125}}{49}$ $(\text{or } 40x^2 - 12x - 9 = 0, \quad x = \frac{12 \pm \sqrt{144 + 3600}}{80})$ $x = 0.647 \dots \text{ (m)} \quad AC \approx 1.4 \text{ (m)}$	M1 A1 A1 DM1 A1 (5)
(c)	Using $F = ma$ and displacement $x$ from $E$ : $0.5g - \frac{24.5(x + 0.15)}{0.75} = 0.5a$ $a = -\frac{196}{3}x, \text{ so SHM}$	M1 A2,1,0 A1
(d)	Max speed = their $a$ x their $\omega$ $= (0.647 - 0.15) \times \sqrt{\frac{196}{3}}$ $\approx 4.0 \text{ ms}^{-1} \quad (4.02)$	M1 (4) A1 (2)
		<b>14</b>



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# Mark Scheme (Results)

January 2013

GCE Mechanics – M3 (6679/01)



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January 2013

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- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Unless indicated in the mark scheme a correct answer with no working should gain full marks for that part of the question.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.

In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\surd$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark.
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* or AG: The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.
  5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.
8. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		•
aA	•	
bM1		•
bA1	•	
bB	•	
bM2		•
bA2		•

**January 2013**  
**6679 M3**  
**Mark Scheme**

Question Number	Scheme	Marks
1.	$v \frac{dv}{dx} = 9x$ $\frac{1}{2}v^2 = 9x \quad (+c)$ $v^2 = 9x^2 + c$ $x = 2 \quad v = 6 \quad 36 = 9 \times 4 + c \Rightarrow c = 0$ $v^2 = 9x^2$	<p>M1</p> <p>A1</p> <p>M1dep</p> <p>A1</p>

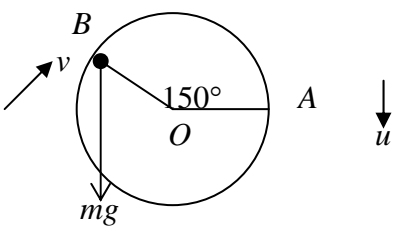
Question Number	Scheme	Marks
<b>2 (a)</b>	<div> <div> <div>Mass:</div> <div> <math>\frac{2}{3}\pi r^3</math> </div> </div> <div> <div> <math>\frac{1}{3}k\pi r^3</math> </div> <div> <math>2+k</math> </div> </div> </div>	B1
	<div> <div> <div>Dist from <math>O</math>:</div> <div> <math>-\frac{3}{8}r</math> </div> </div> <div> <div> <math>\frac{1}{4}kr</math> </div> <div> <math>\bar{x}</math> </div> </div> </div>	B1
	$-\frac{3}{4}r + \frac{k^2 r}{4} = \bar{x}(2+k)$	M1A1ft
	$\bar{x} = \frac{(k^2 - 3)r}{4(k+2)} *$	A1
<b>(b)</b>	$\tan \theta = \frac{(k^2 - 3)r}{4(k+2)} \div r$	M1A1
	$\frac{(k^2 - 3)}{4(k+2)} = \frac{11}{14}$	
	$14k^2 - 42 = 44k + 88$	
	$7k^2 - 22k - 65 = 0$	
	$(7k + 13)(k - 5) = 0$	
	$k = 5$	M1depA1

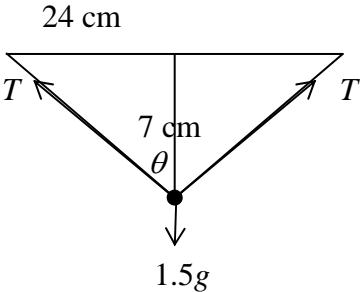
Question Number	Scheme	Marks
<b>3</b>		
<b>(a)</b>	$0.6a = -\frac{12}{(t+2)^2}$ $0.6 \int dv = -\int \frac{12}{(t+2)^2} dt$ $0.6v = \frac{12}{(t+2)} (+c)$ $t=0 \quad v=15 \quad 0.6 \times 15 = 6 + c \Rightarrow c=3$ $0.6v = \frac{12}{(t+2)} + 3 \quad v = \frac{20}{(t+2)} + 5 = 5\left(\frac{4}{t+2} + 1\right) \quad *$	<p>M1</p> <p>M1depA1</p> <p>M1dep</p> <p>A1</p>
<b>(b)</b>	$\frac{dx}{dt} = 5\left(\frac{4}{t+2} + 1\right)$ $x = \int 5\left(\frac{4}{t+2} + 1\right) dt$ $x = 5(4 \ln(t+2) + t) (+c')$ $t=0, x=0 \quad c' = -20 \ln 2$ $t=5 \quad x = 5(4 \ln 7 + 5) - 20 \ln 2$ $= 50.05... = 50.1 \text{ or better}$ $\text{or } 20 \ln\left(\frac{7}{2}\right) + 25$	<p>M1</p> <p>M1depA1</p> <p>M1dep</p> <p>A1</p>

Question Number	Scheme	Marks
<p><b>4</b></p> <p><b>(a)</b></p> <p><b>(b)</b></p>	<div data-bbox="454 342 699 645" data-label="Image"> </div> <p><math>R(\uparrow) \quad T \cos \theta = mg</math></p> <p><math>T \times \frac{2a}{(2a+x)} = mg</math></p> <p>Hooke's Law: <math>T = \frac{6mgx}{2a} = \frac{3mgx}{a}</math></p> <p><math>\frac{3mgx}{a} \times \frac{2a}{(2a+x)} = mg</math></p> <p><math>6x = 2a + x</math></p> <p><math>x = \frac{2}{5}a \quad *</math></p> <p><math>T \sin \theta = \frac{mv^2}{r}</math></p> <p><math>3mg \times \frac{2}{5} \sin \theta = \frac{mv^2}{\left(\frac{12a}{5}\right) \sin \theta}</math></p> <p><math>v^2 = \frac{6}{5}g \times \frac{12a}{5} \sin^2 \theta</math></p> <p><math>\sin^2 \theta = 1 - \left( \frac{4a^2}{\left(\frac{12a}{5}\right)^2} \right) = \frac{11}{36}</math></p> <p><math>v^2 = \frac{72ag}{25} \times \frac{11}{36} = \frac{22ag}{25}</math></p>	<p>M1</p> <p>A1</p> <p>M1A1</p> <p>M1dep</p> <p>A1</p> <p>M1A1</p> <p>M1dep</p> <p>M1depA1</p>



Question Number	Scheme	Marks
<b>5</b>		
<b>(a)</b>	$x = a \sin \omega t$ $0.125 = 0.25 \sin 0.1\omega$ $\sin 0.1\omega = \frac{1}{2}$ $0.1\omega = \frac{\pi}{6}$ $\omega = \frac{\pi}{0.6} = \frac{10\pi}{6}$ $\text{Period} = \frac{2\pi}{\omega} = \frac{6}{5} \quad (=1.2)$	<p>M1A1</p> <p>M1depA1</p> <p>A1 (B1 on e-pen)</p>
<b>(b)</b>	$x = 0.25 \sin \frac{5}{3}\pi t$ $t = 2 \quad x = 0.25 \sin \left( 2 \times \frac{5}{3}\pi \right)$ $x = -0.2165\dots$ $\text{Dist from } B = 0.25 + x = 0.033 \text{ m}$	<p>M1</p> <p>A1 A1 ft</p>
<b>(c)</b>	$\text{Max accel} = a\omega^2 = 0.25 \times \left( \frac{5\pi}{3} \right)^2 = 6.853\dots = 6.85$	M1A1
<b>(d)</b>	$\text{Max speed } a\omega = 0.25 \times \left( \frac{5\pi}{3} \right) = 1.308\dots = 1.31$	M1A1

Question Number	Scheme	Marks
6	 <p>(a) At B <math>mg \cos 60 (+R) = \frac{mv^2}{a}</math></p> <p><math>\frac{1}{2}g = \frac{v^2}{a} \quad v = \sqrt{\frac{ag}{2}} \quad *</math></p> <p>(b) Energy A to B: <math>\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{ag}{2}\right) = mga \sin 30</math></p> <p><math>u^2 = \frac{ag}{2} + 2ag \times \frac{1}{2}</math></p> <p><math>u = \sqrt{\frac{3ag}{2}}</math></p> <p>(c) Horiz speed = <math>\sqrt{\frac{ag}{2}} \cos 60 \left( = \frac{1}{2} \sqrt{\frac{ag}{2}} \right)</math></p> <p>Initial vert speed = <math>(-)\sqrt{\frac{ag}{2}} \sin 60 \left( = (-)\frac{1}{2} \sqrt{\frac{3ag}{2}} \right)</math></p> <p><math>v^2 = \frac{1}{4} \times \frac{3ag}{2} + 2g \times \frac{a}{2}</math></p> <p><math>v^2 = \frac{11ag}{8}</math></p> <p><math>\tan \theta = \frac{\text{vert}}{\text{horiz}} = \sqrt{\frac{11ag}{8} \times \frac{8}{ag}} = \sqrt{11}</math></p> <p><math>\theta = 73.22... = 73</math></p>	<p>M1A1</p> <p>A1</p> <p>M1A1A1</p> <p>A1</p> <p>M1A1</p> <p>M1</p> <p>M1A1</p> <p>M1</p> <p>A1</p>

Question Number	Scheme	Marks
7 (a)	$T = \frac{\lambda x}{l} \Rightarrow 240 = \frac{\lambda \times 18}{30}$	M1A1
	$\lambda = 400$	A1
(b)	 <p>Extension = 10 cm or 20 cm (used in (b) or (c))</p> $T = \frac{400 \times 10}{15} = \left( \frac{800}{3} \right)$ $R(\uparrow) \quad 2T \cos \theta - 1.5g = (\pm) 1.5a$ $\frac{1600}{3} \times \frac{7}{25} - 1.5 \times 9.8 = (\pm) 1.5a$ $a = 89.75 \dots \quad a = 90 \text{ m s}^{-2} \text{ or } 89.8 \text{ (positive)}$	<p>B1</p> <p>M1A1ft</p> <p>M1A1</p>
(c)	$\text{E.P.E.} = \frac{1}{2} \times 400 \times \frac{0.2^2}{0.3}$ $1.5g \times 0.07 + \frac{1}{2} \times 1.5v^2 = 200 \times \frac{0.2^2}{0.3} - \frac{200 \times 0.18^2}{0.3}$ $v^2 = \frac{1}{0.75} \left( 200 \times \frac{0.2^2}{0.3} - \frac{200 \times 0.18^2}{0.3} - 1.5g \times 0.07 \right)$ $v = 2.32 \dots = 2.3 \text{ m s}^{-1}$	<p>B1ft (any correct EPE)</p> <p>M1A1A1</p> <p>M1dep</p> <p>A1</p>

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Mark Scheme (Results)

Summer 2013

GCE Mechanics 3 (6679/01R)

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Question Number	Scheme	Marks
1.	<div data-bbox="292 331 635 712"> </div> <p data-bbox="280 723 395 757">Vertical:</p> $R \cos \beta = mg$ <p data-bbox="280 790 427 824">Horizontal:</p> $R \sin \beta = \frac{mv^2}{r} = \frac{3mv^2}{a}$ <p data-bbox="280 880 379 913">Divide:</p> $\tan \beta = \frac{3mv^2}{amg}$ $\tan \beta = \frac{h}{a}$ $\frac{3mv^2}{amg} = \frac{h}{a}, \quad \frac{3v^2}{g} = h, \quad v = \sqrt{\frac{hg}{3}}$ <p data-bbox="1098 1059 1182 1093">*AG*</p>	<p data-bbox="1286 723 1369 757">M1A1</p> <p data-bbox="1286 768 1369 801">M1A1</p> <p data-bbox="1286 857 1377 891">M1dep</p> <p data-bbox="1286 947 1321 981">B1</p> <p data-bbox="1286 1037 1321 1070">A1</p> <p data-bbox="1441 1070 1481 1137">(7) [7]</p>

Question Number	Scheme	Marks
2.	$F = 1 + 3t^{\frac{1}{2}} = m \frac{dv}{dt} = 4 \frac{dv}{dt}$ $4v = \int 1 + 3t^{\frac{1}{2}} dt = t + 2t^{\frac{3}{2}} (+C)$ $v = \frac{1}{4}(t + 2t^{1.5}) + 2$ $t = 4, v = \frac{1}{4}(4 + 16) + 2 = 7 \text{ (m s}^{-1}\text{)}$ $\text{Work done} = \text{gain in KE} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ $= \frac{1}{2} \times 4 \times 7^2 - \frac{1}{2} \times 4 \times 2^2 = 90 \text{ (J)}$	B1 M1A1 A1 A1ft M1 A1 (7) [7]

Question Number	Scheme	Marks
3. (a)	<p>Weight + thrust = mass x accn.</p> $0.5 \times g + \frac{20 \times 1}{2} = 0.5a$ $a = g + 20 = 29.8 \approx 30 \text{ (m s}^{-2}\text{)}$	<p>M1 B1(thrust) A1ft A1</p> <p>(4)</p>
(b)	<p>Change in GPE = <math>mg(x+1)</math></p> <p>EPE at B = <math>\frac{20 \times 1^2}{2 \times 2}</math> or EPE at C = <math>\frac{20 \times x^2}{2 \times 2}</math></p> <p>Conservation of energy: <math>\frac{20 \times 1^2}{2 \times 2} + mgh = \frac{20 \times x^2}{2 \times 2}</math> <math>h = x + 1</math></p> $5 + 0.5g(x+1) = 5x^2$ $5x^2 - 0.5gx - (5 + .5g) = 0$ $x = \frac{0.5g + \sqrt{(0.5g)^2 + 20(5 + 0.5g)}}{10} = 1.98$ <p>Distance <math>BC = 1 + 1.98 = 2.98 \text{ (m)}</math></p>	<p>B1 B1</p> <p>M1A1</p> <p>M1dep</p> <p>A1</p> <p>(6) [10]</p>

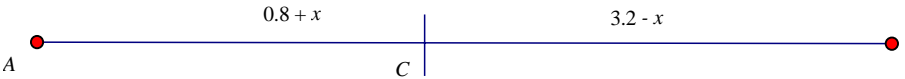
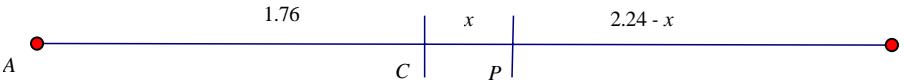
Question Number	Scheme	Marks
4. (a)	$v = \frac{4}{(x+2)} = \frac{dx}{dt}$ $\frac{dt}{dx} = \frac{x+2}{4}; \quad \int_{t=0}^{t=2} 1 dt = \frac{1}{4} \int_{x=0}^{x=X} (x+2) dx, \quad [t]_0^2 = \frac{1}{4} \left[ \frac{x^2}{2} + 2x \right]_0^X$ $2 = \frac{X^2}{8} + \frac{X}{2},$ $0 = X^2 + 4X - 16, \quad X = \frac{-4 + \sqrt{80}}{2} = 2.47 \text{ (m)}$	B1 M1,A1 M1depA1 <b>(5)</b>
(b)	$a \left( = \frac{dv}{dt} \right) = v \frac{dv}{dx}$ $= \frac{4}{(x+2)} \times \frac{-4}{(x+2)^2}$ $= \frac{-16}{(2.47+2)^3} = -0.1788...$ $0.18 \text{ (m s}^{-2}\text{) towards } O.$	B1 M1A1 M1dep A1 <b>(5)</b> <b>[10]</b>

Question Number	Scheme	Marks
5. (a)	<p>Use of Energy at A = energy at B</p> $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh, \quad \frac{1}{2}mgr = \frac{1}{2}mv^2 + mg \times r(1 - \cos \alpha)$ $= \frac{1}{2}mv^2 + mg \times r \times \frac{1}{5}$ $v^2 = gr - \frac{2gr}{5} = \frac{3gr}{5}$ $v = \sqrt{\frac{3gr}{5}} \quad \text{*AG*}$	<p>M1 A1A1</p> <p>A1</p> <p>(4)</p>
(b) v1	<p>Horizontal component of speed at B and at C = their <math>v \cos \theta</math></p> <p>Vertical component of speed at B = their <math>v \sin \theta</math></p> <p>Conservation of energy gives speed at C = <math>\sqrt{\frac{2g}{5}}</math></p> <p>Vertical component of speed at C = <math>\sqrt{\frac{2g}{5} - \frac{16 \times 6g}{25^2}} \approx 1.5539..</math></p> <p><math>v = u + at \Rightarrow t = \frac{1.5539... + 0.92017...}{g} \approx 0.252.. \text{seconds}</math></p> <p>Horizontal distance = <math>\frac{3}{5} \times 0.4 + 1.22689.. \times 0.252... = 0.55 \text{ (m)}</math></p>	<p>M1</p> <p>M1</p> <p>M1A1</p> <p>M1A1</p> <p>M1A1</p> <p>(8)</p>
(b) v2	<p>Horizontal component of speed at B and at C = their <math>v \cos \theta</math></p> <p>Vertical component of speed at B = their <math>v \sin \theta</math></p> $s = ut + \frac{1}{2}at^2 : -\frac{1}{5} \times 0.4 = -\frac{2}{25} = \sqrt{\frac{6g}{25}} \times \frac{3}{5}t - \frac{1}{2}gt^2$ $4.9t^2 - .92017..t - 0.08 = 0$ $t = \frac{0.920 + \sqrt{0.920^2 + 0.32 \times 4.9}}{9.8} = 0.252.....$ <p>Horizontal distance = <math>\frac{3}{5} \times 0.4 + 1.22689.. \times 0.252... = 0.55 \text{ (m)}</math></p>	<p>M1</p> <p>M1</p> <p>M1A1</p> <p>M1A1</p> <p>M1A1</p> <p>(8)</p>

Question Number	Scheme	Marks
(b) v3	Horizontal component of speed at $B$ and at $C = \text{their } v \cos \theta$	M1
	Vertical component of speed at $B = \text{their } v \sin \theta$	M1
	$s = ut + \frac{1}{2}at^2 : -\frac{1}{5} \times 0.4 = -\frac{2}{25} = \sqrt{\frac{6g}{25}} \times \frac{3}{5}t - \frac{1}{2}gt^2$ $4.9t^2 - .92017t - 0.08 = 0$	M1A1
	Horizontal distance from $B = 1.22689... \times t = x$ Form quadratic in $x$ by substituting for $t$ above $3.255x^2 - 0.75x - 0.08 = 0$	M1
	$x = \frac{0.75 + \sqrt{0.75^2 + 4 \times 3.255 \times 0.08}}{2 \times 3.255} = 0.3097...$ Horizontal distance = $\frac{3}{5} \times 0.4 + 0.3097... = 0.55 \text{ (m)}$	M1A1 A1
		(8) [12]

Question Number	Scheme	Marks								
<b>6</b> <b>(a)</b> <b>v1</b>	<p>Mass of quadrant = <math>\rho \frac{\pi a^2}{4}</math></p> $\int_0^a \rho x \sqrt{a^2 - x^2} dx = \rho \left[ -\frac{1}{3} (a^2 - x^2)^{\frac{3}{2}} \right]_0^a$ $= \rho \left[ 0 + \frac{1}{3} a^3 \right]$ $\rho \frac{\pi a^2}{4} \bar{x} = \rho \frac{a^3}{3}$ $\bar{x} = \frac{4a}{3\pi} \quad , \quad \bar{y} = \frac{4a}{3\pi} \text{ by symmetry } \text{ *AG*}$	<p>B1</p> <p>M1A1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1,A1</p> <p><b>(7)</b></p>								
<b>(b)</b>	<table border="1"><tr><td>Area</td><td><math>2a^2</math></td><td><math>\frac{\pi a^2}{4}</math></td><td><math>-\frac{\pi a^2}{4}</math></td></tr><tr><td>Distance to AE</td><td><math>\frac{a}{2}</math></td><td><math>a + \frac{4a}{3\pi}</math></td><td><math>a - \frac{4a}{3\pi}</math></td></tr></table> <p>Moments about AE: <math>2a^2 \bar{x} = 2a^2 \frac{a}{2} + \frac{\pi a^2}{4} (a + \frac{4a}{3\pi}) - \frac{\pi a^2}{4} (a - \frac{4a}{3\pi})</math></p> $= a^3 + \frac{2a^3}{3} = \frac{5a^3}{3}$ $\bar{x} = \frac{5a^3}{3} \times \frac{1}{2a^2} = \frac{5a}{6}$	Area	$2a^2$	$\frac{\pi a^2}{4}$	$-\frac{\pi a^2}{4}$	Distance to AE	$\frac{a}{2}$	$a + \frac{4a}{3\pi}$	$a - \frac{4a}{3\pi}$	<p>B1</p> <p>M1A2</p> <p>A1</p> <p><b>(5)</b></p>
Area	$2a^2$	$\frac{\pi a^2}{4}$	$-\frac{\pi a^2}{4}$							
Distance to AE	$\frac{a}{2}$	$a + \frac{4a}{3\pi}$	$a - \frac{4a}{3\pi}$							
<b>(c)</b>	<p>Taking moments about E: <math>2aX = \frac{5a}{6}W</math></p> $X = \frac{5}{12}W$ <p>their <math>\bar{x}</math></p>	<p>M1A1ft</p> <p>A1</p> <p><b>(3)</b></p>								
<b>6</b> <b>(a)</b> <b>v2</b>	<p>Mass of quadrant = <math>\rho \frac{\pi a^2}{4}</math></p> $\int_0^{\frac{\pi}{2}} \rho \cdot \frac{1}{2} a^2 \cdot \frac{2}{3} a \cos \theta d\theta = \left[ \frac{a^3}{3} \sin \theta \right]_0^{\frac{\pi}{2}} = \rho \frac{a^3}{3}$ $\rho \frac{\pi a^2}{4} \bar{x} = \rho \frac{a^3}{3}$ $\bar{x} = \frac{4a}{3\pi} \quad , \quad \bar{y} = \frac{4a}{3\pi} \text{ by symmetry } \text{ *AG*}$	<p>B1</p> <p>M1A1,=A1</p> <p>M1</p> <p>A1A1</p> <p><b>(7)</b></p> <p><b>[15]</b></p>								



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7 (a)	 <p>Tensions equal when <math>P</math> in equilibrium: <math>\frac{15 \times x}{0.8} = \frac{10 \times (2.4 - x)}{0.8}</math>  <math>25x = 24, \quad x = \frac{24}{25} = 0.96</math>  <math>AC = 1.76 \text{ (m)} \quad \text{*AG*}</math></p>	<p>M1A2</p> <p>A1</p> <p>(4)</p>
(b)	 <p>When <math>P</math> is distance <math>x</math> from <math>C</math>, restoring force  <math>\frac{15 \times (0.96 + x)}{0.8} - \frac{10 \times (1.44 - x)}{0.8} = \frac{25}{0.8} x = -m\ddot{x} = -0.2\ddot{x}</math>  <math>\ddot{x} = -156.25x (= -12.5^2 x) \Rightarrow \text{SHM}</math></p>	<p>M1A2</p> <p>A1</p> <p>(4)</p>
(c)	<p>Speed at <math>C</math> = max speed = <math>a\omega = 0.4 \times 12.5 = 5 \text{ (m s}^{-1}\text{)}</math>  their <math>\omega</math></p>	<p>0.4 x</p> <p>M1A1ft</p> <p>(2)</p>
(d)	<p><math>x = a \cos \omega t</math>  <math>\dot{x} = -a\omega \sin \omega t</math>  <math>(-)^2 = (-)^2 \sin 12.5t</math>  <math>12.5t = 0.4115 \dots, \quad t = 0.0329 \dots \approx 0.033 \text{ (s)}</math></p>	<p>their <math>\omega</math>  their <math>\omega</math>  their <math>\omega</math></p> <p>B1ft  M1  A1ft  A1</p> <p>(4)  [14]</p>

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Mark Scheme (Results)

Summer 2013

GCE Mechanics 3 (6679/01)

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Summer 2013

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1.	$R(\uparrow) \quad R = mg$ $F = \mu mg$ $20 \text{ revs per min} = \frac{20}{60} \times 2\pi \text{ rad s}^{-1}$ $\left( = \frac{2}{3} \pi \text{ rad s}^{-1} \right)$ $R(\rightarrow) \quad \mu mg = m \times 0.4 \times \left( \frac{2}{3} \pi \right)^2$ $\mu = \frac{0.4 \times 4\pi^2}{9g}$ $\mu = 0.18 \text{ or } 0.179$	<p>B1</p> <p>M1A1</p> <p>M1A1ft</p> <p>A1</p> <p><b>[6]</b></p>

#### Notes for Question 1

B1 for resolving vertically and using  $F = \mu R$  to obtain  $F = \mu mg$ . This may not be seen explicitly, but give B1 when seen used in an equation.

M1 for attempting to change revs per minute to  $\text{rad s}^{-1}$ , must see  $(2)\pi$ . (Can use 60 or  $60^2$ )

A1 for  $\frac{20}{60} \times 2\pi \text{ (rad s}^{-1}\text{)}$  oe

M1 for NL2 horizontally along the radius - acceleration in either form for this mark,  $F$  or  $\mu mg$  or  $\mu m$  all allowed.  $r$  to be 0.4 now or later. This is not dependent on the previous M mark.

A1ft for  $\mu mg = m \times 0.4 \times \left( \frac{2}{3} \pi \right)^2$  follow through on their  $\omega$

A1cso for  $\mu = 0.18$  or  $0.179$ , **must be 2 or 3 sf.**

**NB:** Use of  $\leq$ : is allowed, provided used correctly, until the final statement, which must be  $\mu = \dots$



Question Number	Scheme	Marks
2		
(a)	$\left(2t + \frac{1}{2}\right) = 0.5 \frac{dv}{dt}$ $\int (4t + 1) dt = \int dv$ $2t^2 + t = v + c$ $t = 0 \quad v = 0 \quad c = 0$ $v = 2t^2 + t \text{ m s}^{-1}$	<p>M1</p> <p>M1dep <math>c</math> not needed</p> <p>A1 inc the value for <math>c</math> (3)</p>
(b)	$\frac{dx}{dt} = 2t^2 + t$ $x = \frac{2}{3}t^3 + \frac{1}{2}t^2 + k$ $t = 0 \quad x = 0 \quad k = 0$ $x = \frac{2}{3}t^3 + \frac{1}{2}t^2$ $v = 6 \quad 6 = 2t^2 + t \quad 2t^2 + t - 6 = 0$ $(2t - 3)(t + 2) = 0 \quad t = \frac{3}{2}$ $x = \frac{2}{3} \times \left(\frac{3}{2}\right)^3 + \frac{1}{2} \left(\frac{3}{2}\right)^2$ $x = \frac{27}{8} \text{ (oe 3.4, 3.375, 3.38) m}$	<p>M1</p> <p>A1</p> <p>M1A1</p> <p>M1dep</p> <p>A1 cso (6)</p>
		[9]

## Notes for Question 2

(a)

M1 for NL2 with acceleration in the form  $\frac{dv}{dt}$ , seen explicitly or implied by the integration

mass can be 0.5 or  $m$

M1dep for integrating with respect to  $t$  - constant not needed

A1cso for **showing** that  $c = 0$  and giving the final result  $v = 2t^2 + t$  Must see  $t = 0, v = 0$  as a minimum

*By definite integration:*

M1 as above

M1dep for integrating, ignore limits

A1 for substituting the limits 0 and  $v$  and 0 and  $t$  and obtaining  $v = 2t^2 + t$

(b)

M1 for integrating their  $v$  with respect to  $t$  constant not needed

A1 for **showing** that  $k = 0$  If no constant shown this mark is lost.

M1 for setting  $v = 6$  using their answer from (a) **and** attempting to solve the resulting quadratic equation, any valid method. If solved by calculator, **both** solutions must be shown.

A1 for  $t = \frac{3}{2}$  negative solution need not be shown with an algebraic solution

M1dep for using **their** (positive) value for  $t$  to obtain  $x = \dots$  If two positive values were obtained, then allow M1 for substituting either value. Dependent on the first M1 of (b) but not the second.

A1cso for  $x = \frac{27}{8}$  (oe eg 3.375, 3.38) (All marks for (b) must have been awarded)

*By definite integration:*

M1 for integrating their  $v$  with respect to  $t$  limits not needed

A1 for correct integration with lower limits 0.

M1 for setting  $v = 6$  using their answer from (a) **and** attempting to solve the resulting quadratic equation, any valid method. If solved by calculator, **both** solutions must be shown.

A1 for  $t = \frac{3}{2}$  negative solution need not be shown with an algebraic solution

M1dep for substituting **their** limits into **their** integrated  $v$  (sub should be shown). Dependent on the first M1 of (b) but not the second

A1cso for  $x = \frac{27}{8}$  (oe eg 3.375, 3.38)

Question Number	Scheme	Marks
3		
(i)	For $Q$ $T = 2mg$	B1
	For $P$ $T \cos \theta = mg$	M1
	$\cos \theta = \frac{1}{2}$ $\theta = 60^\circ$ *	A1cso
(ii)	For $P \rightarrow$ $T \sin \theta = mr\omega^2$	M1A1
	$2mg \sin \theta = m \times 5l \sin \theta \times \omega^2$	M1depA1
	$\omega^2 = \frac{2g}{5l}$ $\omega = \sqrt{\frac{2g}{5l}}$ *	A1cso
		[8]

### Notes for Question 3

In this question, award marks as though the question is not divided into two parts - ie give marks for equations wherever seen.

(i)

B1 for using  $Q$  (no need to state  $Q$  being used) to state that  $T = 2mg$  or  $T_Q = 2mg$  with  $T_P = T_Q$  seen or implied later.

M1 for attempting to resolve vertically for  $P$   $T$  must be resolved but sin/cos interchange or omission of  $g$  are accuracy errors.

$$mg + 2mg = T + T \cos \theta \text{ gets M0}$$

A1cso for combining the two equations to obtain  $\theta = 60^\circ$  \*

**NB:** This is a "show" question, so if no expression is seen for  $T$  and just  $2mg \cos \theta = mg$  shown, award 0/3 as this equation could have been produced from the required result, so insufficient working.

(ii)

M1 for attempting NL2 for  $P$  along the radius. The mass used must be  $m$  if the particle is not stated to be  $P$ ; a mass of  $2m$  would imply use of  $Q$ .  $T$  must be resolved. Acceleration can be in either form.

$$A1 \text{ for } T \sin \theta = m r \omega^2 \text{ or } T \frac{\sqrt{3}}{2} = m r \omega^2$$

M1 dep for eliminating  $T$  between the two equations for  $P$  and substituting for  $r$  in terms of  $l$  and  $\theta$  dependent on the second but not the first M mark.

$$A1 \text{ for } 2mg \sin \theta = m \times 5l \sin \theta \times \omega^2 \text{ or } \frac{T \sin \theta}{T \cos \theta} = \tan \theta = 5l \sin \theta \left( \frac{\omega^2}{g} \right) \theta \text{ or } 60^\circ$$

$$A1cso \text{ for re-arranging to obtain } \omega = \sqrt{\frac{2g}{5l}} \quad * \quad \text{ensure the square root is correctly placed}$$

*Alternatives:* Some candidates "cancel" the  $\sin \theta$  without ever showing it.

$$M1A1 \text{ for } T = m \times 5l \omega^2$$

$$M1A1 \text{ for } 2mg = 5ml \omega^2$$

A1cso as above

**Vector Triangle method: Triangle must be seen**

$$T = 2mg \quad B1$$

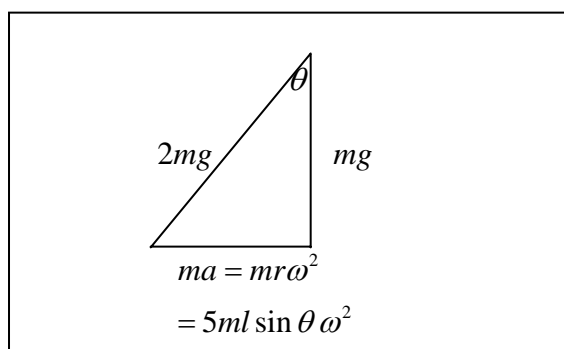
$$\cos \theta = \frac{mg}{2mg} \quad M1$$

$$\theta = 60^\circ \quad A1$$

Correct triangle  $M1A1$

$$\sin \theta = \frac{5ml \sin \theta \omega^2}{2mg} \quad M1A1$$

$$\omega = \dots \quad A1cso \text{ (as above)}$$



Question Number	Scheme	Marks
4		
(a)	$T = \frac{\lambda x}{l}$ $20 = \frac{\lambda \times 0.3}{1.2}$ $\lambda = 80 \text{ N}$ $\text{Initial EPE} = \frac{\lambda x^2}{2l} = \frac{80 \times 0.3^2}{2.4} (= 3 \text{ J})$ $\frac{80 \times 0.3^2}{2.4} - 0.4 \times 2g \times 0.3 = \frac{1}{2} \times 2v^2$ $v^2 = 0.648$ $v = 0.80 \text{ or } 0.805 \text{ m s}^{-1}$	M1A1  A1  B1  M1A1ft    A1 (7)
(b)	<p>Comes to rest <math>0.4 \times 2g \times y = 3</math></p> $y = \frac{3}{0.4 \times 2 \times 9.8} = 0.38 \text{ or } 0.383 \text{ m}$ <p><i>Alternatives:</i> Energy from string going slack to rest:</p> $\frac{1}{2} \times 2 \times 0.648 = 0.4 \times 2g \times x$ $x = 0.8265\dots$ $y = 0.3 + 0.08265\dots = 0.38 \text{ or } 0.383$ <p>NL2 to obtain the accel when string is slack <math>\left(-\frac{2g}{5}\right)</math> and <math>v^2 = u^2 + 2as</math></p> $0 = 0.648 + 2 \times \left(-\frac{2g}{5}\right)s$ $BC = \frac{0.648 \times 5}{4g} + 0.3 = 0.38 \text{ or } 0.383$	M1  A1 (2)  <b>[9]</b>          M1 Complete method A1          M1A1

### Notes for Question 4

**(a)**

M1 for attempting Hooke's Law, formula must be correct, either explicitly or by correct substitution.

A1 for  $20 = \frac{\lambda \times 0.3}{1.2}$

A1 for obtaining  $\lambda = 80$

B1 for the initial EPE  $\frac{\lambda \times 0.3^2}{2.4}$  (= 3 J) their value for  $\lambda$  allowed. May only be seen in the equation.

M1 for a work-energy equation with one EPE term, one KE term and work done against friction (Award if second EPE/KE terms included **provided** these become 0). The EPE must be dimensionally correct, but need not be fully correct (eg denominator 1.2 instead of 2.4)

A1ft for a completely correct equation follow through their EPE

A1 cao for  $v = 0.80$  or  $0.805$  must be 2 or 3 sf

**NB:** This is damped harmonic motion (due to friction) so all SHM attempts lose the last 4 marks.

**(b)**

M1 for any **complete** method leading to a value for either  $BC$ . If the distance travelled after the string becomes slack is found the work must be completed by adding 0.3 Their EPE found in (a) used in energy methods.

**MS** method is energy from  $B$  to  $C$  ie work done against friction = loss of EPE.

**OR** Energy from point where the string becomes slack to  $C$  ie work done against friction = loss of KE and completed for the required distance

**OR** NL2 to obtain the acceleration  $\left(-\frac{2g}{5}\right)$  while the string is slack **and**  $v^2 = u^2 + 2as$  to find the distance and completed for the required distance

A1cso for  $BC = 0.38$  or  $0.383$  (m) **must be 2 or 3 sf**

Question Number	Scheme	Marks
5(a)	$V = \int_0^2 \pi y^2 dx = \pi \int_0^2 (x+1)^4 dx$ $= \pi \left[ \frac{1}{5} (x+1)^5 \right]_0^2$ $= \frac{1}{5} \pi [3^5 - 1] \quad \left( = \frac{242\pi}{5} \right)$	M1 A1 M1
	$\int_0^2 \pi y^2 x dx = \pi \int_0^2 x(x+1)^4 dx$ $= \pi \left[ \frac{x(x+1)^5}{5} \right]_0^2 - \pi \int_0^2 \frac{(x+1)^5}{5} dx, = \frac{2 \times 3^5 \pi}{5} - \pi \left[ \frac{(x+1)^6}{30} \right]_0^2$ $\left[ \frac{2 \times 3^5}{5} - \frac{3^6}{30} + \frac{1}{30} \right] \pi \quad (= 72.933... \pi)$	M1 A1 M1
	<b>ALT:</b> by expanding $= \pi \int_0^2 (x^5 + 4x^4 + 6x^3 + 4x^2 + x) dx$ $= \pi \left[ \frac{x^6}{6} + \frac{4}{5} x^5 + \frac{6}{4} x^4 + \frac{4}{3} x^3 + \frac{1}{2} x^2 \right]_0^2$ $= \pi \left[ \frac{2^6}{6} + \frac{4}{5} \times 2^5 + \frac{6}{4} \times 2^4 + \frac{4}{3} \times 2^3 + \frac{1}{2} \times 2^2 \right]$	M1A1 M1
	<b>OR by subst:</b> $\pi \int_1^3 (u-1) u^4 du, = \pi \left[ \frac{u^6}{6} - \frac{u^5}{5} \right]_1^3, = \pi \left[ \frac{3^6}{6} - \frac{3^5}{5} - \left( \frac{1}{6} - \frac{1}{5} \right) \right]$	M1A1M1
(b)	$\bar{x} = \frac{\pi \left[ \frac{2 \times 3^5}{5} - \frac{3^6 - 1}{30} \right]}{\frac{242\pi}{5}} \text{ OR } \frac{\pi \left[ \frac{2^6}{6} + \frac{4 \times 2^5}{5} + \frac{6 \times 2^4}{4} + \frac{4 \times 2^3}{3} + \frac{2^2}{2} \right]}{\frac{242\pi}{5}}, = 1.5068$ <p style="text-align: center;">hemisphere                      S                      T</p> <p>Mass ratio                      <math>10 \times \frac{2\pi}{3} \times 1</math>                      <math>\frac{242\pi}{5}</math>                      <math>\left( \frac{20}{3} + \frac{242}{5} \right) \pi = \frac{826}{15} \pi</math></p> <p>Dist from A                      <math>2 + \frac{3 \times 1}{8}</math>                      0.493                      <math>\bar{x}</math></p> $\frac{20}{3} \times \frac{19}{8} + \frac{242}{5} \times 0.493 = \left( \frac{20}{3} + \frac{242}{5} \right) \bar{x}$ <p><math>\bar{x} = 0.7208... \text{ cm}</math> (awrt 0.72)</p>	M1, A1 (8)  B1ft on S  B1ft on S  M1A1ft  A1 (5) <b>[13]</b>

### Notes for Question 5

**NB:** Some candidates will omit  $\pi$  throughout (as they know it cancels). In such cases award all marks if earned. If  $\pi$  is omitted from one integration only but then appears in the result of that integration at the last stage or is then omitted from the second integration, all marks can be gained. But if omitted from one integration, including the last stage, and included with the other mark strictly according to the MS.

(a)

M1 for using  $V = \int_0^2 \pi y^2 dx = \pi \int_0^2 (x+1)^4 dx$  - limits not needed and attempting the integration by inspection or expansion (algebra **must** be seen)

A1 for correct integration - limits not needed

M1 for substituting the correct limits into **their** integrated function - no need to simplify

M1 for attempting to integrate  $\int_0^2 \pi y^2 x dx = \pi \int_0^2 x(x+1)^4 dx$  - limits not needed - by parts. This mark can be awarded once the integral has been expressed as the difference of an appropriate integrated function and an integral

A1 for correct, complete integration  $\pi \left[ \frac{x(x+1)^5}{5} \right]_0^2 - \pi \left[ \frac{(x+1)^6}{30} \right]_0^2$  or  $\frac{2 \times 3^5 \pi}{5} - \pi \left[ \frac{(x+1)^6}{30} \right]_0^2$  Limits not needed

M1 for substituting the correct limits into **their** integrated function - no need to simplify

Alternative methods for  $\int_0^2 \pi y^2 x dx = \pi \int_0^2 x(x+1)^4 dx$

M1 for expanding and integrating or making a suitable substitution and attempting the integration - limits not needed

A1 for correct integration - limits not needed

M1 for substituting the correct limits into **their** integrated function - no need to simplify

M1 for using  $\bar{x} = \frac{\int \pi y^2 x dx}{\int \pi y^2 dx}$  Their integrals need not be correct.

A1cao for  $\bar{x} = 1.5068...$  Accept 1.5, 1.51 or better or  $\frac{547}{363}$

(b)

B1ft for correct mass ratio, follow through their volume for  $S$  need  $\pi$  now

B1ft for correct distances, follow through their distance for  $S$ , but remember it must be 2 - answer from (a) if working from A. Distances from the common face are  $-\frac{3}{8}$ , ans from (a),  $\bar{x}$  Distances from other end are  $\frac{5}{8}$ , 1+ ans from (a),  $\bar{x}$

M1 for a dimensionally correct moments equation

A1ft for a fully correct moments equation, follow through their distances and mass ratio

A1cao for 0.7208... Accept 0.72 or better (Exact is  $\frac{1191}{1652}$ )



Question Number	Scheme	Marks
6(a)	$\frac{24e}{1.5} = \frac{18(1.5 - e)}{0.75}$ $16e = 36 - 24e$ $e = 0.9$ $AO = 2.4 \text{ m}^*$	M1A1   A1  A1ft (4)
(b)	$\frac{18(0.6 - x)}{0.75} - \frac{24(0.9 + x)}{1.5} = m\ddot{x} \text{ or } 0.8\ddot{x}$ $14.4 - 24x - 14.4 - 16x = m\ddot{x} \text{ or } 0.8\ddot{x}$ $\ddot{x} = -\frac{40x}{0.8 \text{ or } m} (= -50x) \therefore \text{SHM}$	M1A1A1   M1depA1 (5)
(c)	$\ddot{x} = -50x \Rightarrow \omega = \sqrt{50} \text{ or } 5\sqrt{2}$ $\text{max. speed} = \sqrt{2} \Rightarrow a \times 5\sqrt{2} = \sqrt{2}$ $a = \frac{1}{5}$ $-0.1 = 0.2 \cos(5\sqrt{2})t$ $t = \frac{1}{5\sqrt{2}} \cos^{-1}\left(-\frac{1}{2}\right)$ $t = \frac{1}{5\sqrt{2}} \times \frac{2\pi}{3} = \frac{\pi\sqrt{2}}{15} \text{ or } 0.296 \text{ s } (0.2961...) \text{ Accept } 0.30, \text{ or better}$	B1  M1  A1  M1  A1 (5)
		[14]

## Notes for Question 6

(a)

M1 for using Hooke's Law for each string, equating the two tensions and solving to find the extension in either string. The extensions should add to 1.5. The formula for Hooke's law must be correct, either shown explicitly in its general form or implicitly by the substitution.

A1 for a correct equation

A1 for  $e = 0.9$

A1cso for 2.4 (m) \*

*Alternative:* Find the ratio of the two extensions and divide 1.5 m in that ratio.

M1 complete method      A1 correct ratio      A1 extension in AO

A1 2.4 (m)

(b)

M1 for an equation of motion for  $P$ . There must be a difference of two tensions. The acceleration can be  $a$  or  $\ddot{x}$  here and  $x$  should be measured from the equilibrium position ( $O$ ) unless a suitable substitution is made later. Mass can be  $m$  or 0.8

A1,A1 for  $\frac{18(0.6-x)}{0.75} - \frac{24(0.9+x)}{1.5} = m\ddot{x}$  or  $0.8\ddot{x}$  or  $a$  instead of  $\ddot{x}$  Give A1A1 if the equation is completely correct and A1 if only one error. Note that if the difference of the tensions is the wrong way round, this is *one* error

M1dep for simplifying to  $\ddot{x} = f(x)$  Must be  $\ddot{x}$  now.

A1 for  $\ddot{x} = -\frac{40x}{0.8 \text{ or } m}$  ( $= -50x$ ) **and the conclusion** (ie  $\therefore$  SHM)

(c)

B1 for  $\omega = \sqrt{50}$  or  $5\sqrt{2}$  need not be shown explicitly

M1 for using max speed  $= a\omega = \sqrt{2}$  with **their**  $\omega$

A1 for  $a = \frac{1}{5}$

M1 for using  $x = a \cos \omega t$  with **their**  $\omega$  and  $a$  and  $x = \pm(0.3 - a)$  **or**  $x = a \sin \omega t$  provided the work is completed by adding a quarter of their period is added to the time to complete the method.

A1cao for  $t = \frac{\pi\sqrt{2}}{15}$  or 0.296s (0.2961...) Accept 0.30 or better

Question Number	Scheme	Marks
<b>7</b>	$T - 5mg \cos \theta = \frac{5mv^2}{a}$	M1A1
<b>(a)</b>	$\frac{1}{2} \times 5mv^2 - \frac{1}{2} \times 5m \times \frac{9ag}{5} = 5mga \cos \theta$	M1A1
	$5mv^2 = 10mga \cos \theta + 9mga$	
	$T = 5mg \cos \theta + 10mg \cos \theta + 9mg$	M1dep
	$T = 3mg(5 \cos \theta + 3) \quad *$	A1 (6)
<b>(b)</b>	$T = 0 \quad \cos \theta = -\frac{3}{5}$	B1
	$v^2 = \frac{9ag}{5} - \frac{6ag}{5} = \frac{3ag}{5}$	M1
	$v = \sqrt{\frac{3ag}{5}}$	A1 (3)
<b>(c)</b>	horiz comp of vel at B = $\sqrt{\frac{3ag}{5}} \times \frac{3}{5}$	M1
	vert comp = $\sqrt{\frac{3ag}{5}} \times \frac{4}{5}$	M1
<b>(i)</b>	$x = -\frac{4a}{5} + \frac{3}{5} \sqrt{\frac{3ag}{5}} t$	M1depA1
	$y - \frac{3a}{5} = \frac{4}{5} \sqrt{\frac{3ag}{5}} t - \frac{1}{2} gt^2$	M1depA1ft
<b>(ii)</b>	$y = \frac{4}{5} \sqrt{\frac{3ag}{5}} t - \frac{1}{2} gt^2 + \frac{3a}{5}$	A1 (7)
		[16]

### Notes for Question 7

**(a)**

M1 for attempting NL2 along the radius when the string makes an angle  $\theta$  with the downward vertical. The acceleration can be in either form, the weight must be resolved and  $T$  must be included (not resolved). Sin/cos interchange or omission of  $g$  are accuracy errors as is omission of 5 in one or both terms. Radius can be  $a$  or  $r$ .

A1 for a correct equation  $T - 5mg \cos \theta = \frac{5mv^2}{a}$  Acceleration must be in the  $\frac{v^2}{r}$  form now.

M1 for a conservation of energy equation from the horizontal to the same point. There must be a difference of 2 KE terms and a loss of PE term (which may be indicated by a difference of 2 PE terms). The initial KE can be  $\frac{1}{2} \times \text{mass} \times \left( \sqrt{\frac{9ag}{3}} \right)^2$  or  $\frac{1}{2} \times \text{mass} \times u^2$  for this mark. Omission of  $g$  and sin/cos interchange are accuracy errors. Mass can be  $m$  or  $5m$  here or just "mass". Use of  $v^2 = u^2 + 2as$  gets M0

A1 for a fully correct equation  $\frac{1}{2} \times (5m)v^2 - \frac{1}{2} \times (5m) \times \frac{9ag}{5} = (5m)ga \cos \theta$

M1dep for eliminating  $v^2$  between the 2 equations. Dependent on both previous M marks.

A1cso for  $T = 3mg(5 \cos \theta + 3)$  \*

**(b)**

B1 for obtaining  $\cos \theta = -\frac{3}{5}$

M1 for using **their** value for  $\cos \theta$  - must be numerical - in the energy equation to get  $v^2 = \dots$  (no need to simplify) Accept with  $5m$  or  $m$ .

**OR** making  $T = 0$  and  $\cos \theta = -\frac{3}{5}$  (their value) in  $T - 5mg \cos \theta = \frac{5mv^2}{a}$

A1cao for  $v = \sqrt{\frac{3ag}{5}}$  oe Check square root is applied correctly.

**(c)**

M1 for resolving **their**  $v$  to get the horizontal component of the speed at  $B$ . May not be seen explicitly, but seen in their attempt at  $x$ .

M1 for resolving **their**  $v$  to get the vertical component of the speed at  $B$

Both of these M marks can be given if sin and cos are interchanged or numerical substitutions not made.

M1dep for attempting to obtain  $x$  by using the distance from  $B$  to the  $y$ -axis with the horizontal distance travelled (found using their horizontal component, so dependent on the first M1 of (c))

A1cso for  $x = -\frac{4a}{5} + \frac{3}{5} \sqrt{\frac{3ag}{5}} t$

### Notes for Question 7 Continued

M1dep for attempting to obtain  $y$  by using  $s = ut + \frac{1}{2}at^2$  with **their** vertical component and using the initial vertical distance above the  $x$ -axis. Dependent on the second M mark of (c)

A1ft for  $y - \frac{3a}{5} = \frac{4}{5}\sqrt{\frac{3ag}{5}}t - \frac{1}{2}gt^2$  Follow through their initial vertical component

A1cao for  $y = \frac{4}{5}\sqrt{\frac{3ag}{5}}t - \frac{1}{2}gt^2 + \frac{3a}{5}$

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# Mark Scheme (Results)

January 2014

Pearson Edexcel International  
Advanced Level

Mechanics 3 (WME03/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  7. Ignore wrong working or incorrect statements following a correct answer.

**General Notes From Chief Examiner**

- Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra  $g$  in a resolution is accuracy error not method error.
- Omission of mass from a resolution is method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of  $g = 9.8$  should be given to 2 or 3 SF.
- Use of  $g = 9.81$  should be penalised once per (complete) question.
- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *ONCE* per complete question.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.

Question Number	Scheme	Marks				
1.	$v = \sqrt{\left(8x^{\frac{3}{2}} - 4\right)}$ $v^2 = \left(8x^{\frac{3}{2}} - 4\right)$ $2v \frac{dv}{dx} = 12x^{\frac{1}{2}}$ $F = 0.5 \times 6x^{\frac{1}{2}} = 3x^{\frac{1}{2}}$ $x = 4 \Rightarrow F = 6$	M1 A1 A1  M1dep A1 5				
Notes						
<p>M1 for attempting to differentiate the expression for <math>v^2</math> - chain rule must be used on lhs.</p> <p>A1 for correct <math>x^{\frac{1}{2}}</math></p> <p>A1 for 6 Award both only if work fully correct</p> <p>M1dep for using NL2 with <math>m = 0.5</math> to obtain an expression for <math>F</math> in terms of <math>x</math></p> <p>A1cso for <math>F = 6</math></p> <p>Alternatives: for the first 3 marks</p> <table><tr><td><math display="block">\frac{dv}{dx} = \frac{1}{2} \left(8x^{\frac{3}{2}} - 4\right)^{-\frac{1}{2}} \times 12x^{\frac{1}{2}}</math> <math display="block">\frac{dv}{dx} = \frac{1}{2v} \times 12x^{\frac{1}{2}} \quad v \frac{dv}{dx} = 6x^{\frac{1}{2}}</math></td><td>M1 Must be a complete method to obtain accel in terms of <math>x</math> A1rhs A1lhs</td></tr><tr><td><math display="block">\frac{dv}{dt} = \frac{1}{2} \left(8x^{\frac{3}{2}} - 4\right)^{-\frac{1}{2}} \times 12x^{\frac{1}{2}} \times \frac{dx}{dt}</math> <math display="block">\frac{dv}{dt} = \frac{1}{2} \left(8x^{\frac{3}{2}} - 4\right)^{-\frac{1}{2}} \times 12x^{\frac{1}{2}} \times \left(8x^{\frac{3}{2}} - 4\right)^{\frac{1}{2}} = 6x^{\frac{1}{2}}</math></td><td>M1A1A1 Award as above</td></tr></table>		$\frac{dv}{dx} = \frac{1}{2} \left(8x^{\frac{3}{2}} - 4\right)^{-\frac{1}{2}} \times 12x^{\frac{1}{2}}$ $\frac{dv}{dx} = \frac{1}{2v} \times 12x^{\frac{1}{2}} \quad v \frac{dv}{dx} = 6x^{\frac{1}{2}}$	M1 Must be a complete method to obtain accel in terms of $x$ A1rhs A1lhs	$\frac{dv}{dt} = \frac{1}{2} \left(8x^{\frac{3}{2}} - 4\right)^{-\frac{1}{2}} \times 12x^{\frac{1}{2}} \times \frac{dx}{dt}$ $\frac{dv}{dt} = \frac{1}{2} \left(8x^{\frac{3}{2}} - 4\right)^{-\frac{1}{2}} \times 12x^{\frac{1}{2}} \times \left(8x^{\frac{3}{2}} - 4\right)^{\frac{1}{2}} = 6x^{\frac{1}{2}}$	M1A1A1 Award as above	
$\frac{dv}{dx} = \frac{1}{2} \left(8x^{\frac{3}{2}} - 4\right)^{-\frac{1}{2}} \times 12x^{\frac{1}{2}}$ $\frac{dv}{dx} = \frac{1}{2v} \times 12x^{\frac{1}{2}} \quad v \frac{dv}{dx} = 6x^{\frac{1}{2}}$	M1 Must be a complete method to obtain accel in terms of $x$ A1rhs A1lhs					
$\frac{dv}{dt} = \frac{1}{2} \left(8x^{\frac{3}{2}} - 4\right)^{-\frac{1}{2}} \times 12x^{\frac{1}{2}} \times \frac{dx}{dt}$ $\frac{dv}{dt} = \frac{1}{2} \left(8x^{\frac{3}{2}} - 4\right)^{-\frac{1}{2}} \times 12x^{\frac{1}{2}} \times \left(8x^{\frac{3}{2}} - 4\right)^{\frac{1}{2}} = 6x^{\frac{1}{2}}$	M1A1A1 Award as above					

Question Number	Scheme	Marks
2	$\frac{2mg}{2l} \left( \left( \frac{1}{2}l \right)^2 - x^2 \right) = \frac{1}{4}mg \left( \frac{1}{2}l + x \right)$ $8x^2 + 2lx - l^2 = 0$ $(4x - l)(2x + l) = 0$ $x = \frac{1}{4}l \text{ or } -\frac{1}{2}l$ $\text{distance} = \frac{1}{2}l + \frac{1}{4}l = \frac{3}{4}l$	M1A1;M1 A 1 M1 A1 M1dep A1 A1 <b>9</b>
<b>Notes</b>		
M1	for the difference of 2 elastic energy terms, not nec in a complete energy equation.	
A1	for a correct difference	
M1	for a work energy equation, loss of EPE = work done against friction(not dep on previous mark)	
A1	for a fully correct equation	
M1dep	for re-arranging to a three term quadratic, dependent on the second M mark, or use the difference of 2 squares to get a linear equation	
A1	for a correct 3 term quadratic, terms in any order	
M1dep	for solving the resulting quadratic, usual rules. Dependent on all second and third M marks	
A1	for $x = \frac{1}{4}l$ $x = -\frac{1}{2}l$ need not be shown	
A1cao and cso	distance $= \frac{3}{4}l$	

Question Number	Scheme	Marks
3	$\frac{9}{8}mg - mg = \frac{mu^2}{2a}$ $u^2 = \frac{ag}{4}$ $\frac{1}{2}m\left(\frac{ag}{4}\right) - \frac{1}{2}m\left(\frac{ag}{20}\right) = mg2a(1 - \cos\theta)$ $\theta = 18^\circ \text{ nearest degree}$	M1 A1 A1  M1 A1 A1  M1dep A1 <b>8</b>

**Notes**

- M1 for NL2 along the radius at the bottom or top. Must have 2 forces and an acceleration
- A1 for a fully correct equation ie  $\frac{9}{8}mg - mg = \frac{mu^2}{2a}$  oe Must be at the bottom
- A1 for obtaining  $u^2 = \frac{ag}{4}$
- M1 for an energy equation from the bottom or top to the point where the speed is  $\sqrt{\frac{ag}{20}}$  (this may be  $v$  here and for the 2A marks). Must have a difference of KE terms and a GPE term.
- A1ft for correct difference of KE terms or correct PE term (from bottom)  
Follow through their  $u$ .
- A1 for a completely correct equation
- M1dep for substituting  $v = \sqrt{\frac{ag}{20}}$  and solving for  $\theta$  Dependent on both previous M marks
- A1cao  $\theta = 18^\circ$  **must be nearest degree.**

If candidates do the energy equation first, give those 3 marks for an equation with  $u$  (speed at bottom) and  $\sqrt{\frac{ag}{20}}$ . The final M mark will then be for substituting  $u^2 = \frac{ag}{4}$  and solving for  $\theta$ .

If the radius is  $a$  throughout, treat as mis-read. If sometimes  $a$  and sometimes  $2a$  mark each equation on it own merit.

Question Number	Scheme	Marks
<b>4(a)</b>	$\pi \int_0^1 e^{-2x} dx = \frac{\pi}{-2} [e^{-2x}]_0^1$ $= \frac{\pi}{2} (1 - e^{-2}) \quad \text{PRINTED ANSWER}$	<p>M1 A1</p> <p>A1cs0 3</p>
<b>(b)</b>	$\pi \int_0^1 x e^{-2x} dx = \pi \left[ \frac{-1}{2} x e^{-2x} \right]_0^1 - \pi \int_0^1 \frac{-1}{2} e^{-2x} dx$ $= \pi \left( -\frac{1}{2} e^{-2} + \frac{1}{2} \left[ -\frac{1}{2} e^{-2x} \right]_0^1 \right)$ $= \pi \left( -\frac{1}{2} e^{-2} - \frac{1}{4} (e^{-2} - 1) \right)$ $= \pi \left( \frac{1}{4} - \frac{3}{4} e^{-2} \right)$ $\bar{x} = \frac{\pi \left( \frac{1}{4} - \frac{3}{4} e^{-2} \right)}{\frac{\pi}{2} (1 - e^{-2})} = \frac{1}{2} \frac{(e^2 - 3)}{(e^2 - 1)}$	<p>M1 A1</p> <p>M1dep A1ft</p> <p>A1cao</p> <p>M1 A1</p> <p>(7)</p>

**10**

## Notes for Question 4

A note about  $\pi$ : (a) is a "show that" so  $\pi$  must be included throughout (unless a put in at the end of (a), with a convincing argument for doing so). No answer given in (b), so allow the first 5 marks (as earned) without  $\pi$  **provided** either no  $\pi$ s or both  $\pi$ s appear for the final 2 marks. If the final fraction has the denominator  $\pi$  only, the last 3 marks will be lost

(a)

M1 for using  $V = \pi \int y^2 dx = \pi \int e^{-2x} dx$  and attempting the integration. limits not needed for this mark

A1 for correct integration, correct limits must be shown

A1cso for  $V = \frac{\pi}{2}(1 - e^{-2})$  \* Must be seen in this form

(b)

M1 for attempting the integration of  $\pi \int x e^{-2x} dx$  by parts - limits not needed yet. Allow if intention to integrate  $\pi \int x y^2 dx$  is shown.

A1 for a correct result with or w/o limits (check signs carefully)

M1dep for attempting the next integral, limits not needed

A1 ft for substituting the correct limits in their integral

A1cao for  $\pi \left( \frac{1}{4} - \frac{3}{4} e^{-2} \right)$  oe

M1 for using  $\bar{x} = \frac{(\pi) \int x y^2 dx}{(\pi) \int y^2 dx}$  with their integrals, must be the correct way up.

A1 for  $\bar{x} = \frac{(e^2 - 3)}{2(e^2 - 1)}$  oe **must be in terms of e**. Must have only 2 terms in each of the numerator and denominator and no fractions in either.



Question Number	Scheme	Marks
<b>5(a)</b>	$3k\frac{2}{3}\pi r^3 \quad k\pi r^2 3r \quad 3k\frac{2}{3}\pi r^3 + k\pi r^2 3r$ $(2) \quad (3) \quad (5)$ $\left(\frac{3r}{8} + 3r\right) \quad \frac{3r}{2} \quad \bar{x}$ $\left(\frac{3r}{8} + 3r\right).2 + \frac{3r}{2}.3 = 5\bar{x}$ $\frac{9r}{4} = \bar{x} \quad \text{PRINTED ANSWER}$	<p>B1</p> <p>B1</p> <p>M1 A1ft</p> <p>A1 (5)</p>
<b>(b)</b>	$R = W ; F = P$ $P.2r \sin \alpha = W\left(\frac{9r}{4} \sin \alpha - r \cos \alpha\right)$ $P = W\left(\frac{9}{8} - \frac{1}{2} \cot \alpha\right)$ $F = \mu R$ $\frac{1}{8}(9 - 4 \cot \alpha) = \mu \quad \text{PRINTED ANSWER}$	<p>B1</p> <p>M1 A1 A1</p> <p>A1</p> <p>M1depA1cso (7) <b>12</b></p>

### Notes

**(a)** B1 for a correct ratio of masses

B1 for correct distances of the c of ms of the two components, hopefully from  $O$ , but can be from another point

M1 for a moments equation about  $O$  or their chosen point. Must have three terms and be dimensionally correct

A1ft for a correct equation, follow through their ratio of masses and distances, but **not** 1:3:4 (from mass/unit vol)

A1cso for  $\bar{x} = \frac{9r}{4}$  \*

Special case: Using volumes: max B0B1M1A1A1

**(b)**B1 for the two shown equations

M1 for a moments equation about the point of contact

A1A1 Award A2 if eqn fully correct; A1A0 if one error

A1 for re-arranging to obtain  $P = W\left(\frac{9}{8} - \frac{1}{2} \cot \alpha\right)$

M1dep for using  $F = \mu R$  together with the expression for  $P$  and the first two equations to obtain an expression for  $\mu$

A1cso for  $\mu = \frac{1}{8}(9 - 4 \cot \alpha)$  \* must be this form

Question Number	Scheme	Marks
<b>6(a)</b>	$(6a)^2 + (8a)^2 = (10a)^2$	M1
	by Pythag (converse), $\angle APB = 90^\circ$ PRINTED ANSWER	A1 (2)
<b>(b)</b>	$T_1 \sin \alpha + T_2 \cos \alpha = m r \omega^2$	M1 A2
	$T_1 \cos \alpha - T_2 \sin \alpha = m g$	M1 A1
	$r = 8a \sin \alpha$	M1 A1
	$\sin \alpha = \frac{3}{5} \quad \text{or} \quad \cos \alpha = \frac{4}{5}$	B1
	solving, $T_2 = \frac{3m}{25}(32a\omega^2 - 5g)$	M1
	$T_2 \geq 0 \Rightarrow \omega = \sqrt{\frac{5g}{32a}}$	M1 A1
	$\text{max time} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{32a}{5g}}$ PRINTED ANSWER	M1A1 (13) <b>15</b>

## Notes for Question 6

(a)

M1 for squaring the sides and showing they fit Pythagoras' theorem or ratio of sides 3:4:5 or use the cosine rule

A1cso for stating that (the converse of) Pythagoras' theorem shows that  $APB = 90^\circ$  \* or appropriate conclusion for their method

(b)

M1 for NL2 horizontally. There must be two tensions, both resolved, but may be the same, and an acceleration (either form accepted here) Sine/cos interchange is an accuracy error.

A1 for any two correct terms

A1 for the third correct term. Acceleration must be in the form  $m\omega^2$  and tensions must be different for both these marks to be awarded

M1 for resolving vertically. Again, two tensions, both resolved but may be the same, and sine/cos interchange is an accuracy error.

A1 for a fully correct equation with different tensions.

M1 for finding the radius as  $r = 8a \sin \alpha$  or  $8a \cos \alpha$

A1 for  $r = 8a \sin \alpha$  May not be shown explicitly

B1 for a correct value for  $\sin \alpha$  or  $\cos \alpha$

M1dep for solving to obtain an expression for  $T_2$  in terms of  $m, g, a, \omega$ . Dependent on all M marks above **and** two different tensions. Or making  $T_2 = 0$  in the above equations and solving for  $\omega$

M1dep for using  $T_2 \geq 0$  in *their* expression for  $T_1$  to obtain an expression for  $\omega$  in terms of  $g$  and  $a$   
Dependent on the previous M mark  $T_2 < 0$  gets M0

A1 for  $\omega_{\min} = \sqrt{\frac{5g}{32a}}$  oe

M1 for using  $\frac{2\pi}{\omega}$  with their  $\omega$  to obtain the maximum time

A1cso for max time =  $2\pi \sqrt{\frac{32a}{5g}}$

Question Number	Scheme	Marks
<b>7 (a)</b>	$\frac{8mge}{l} = mg$ $e = \frac{1}{8}l$	M1 A1 (2)
<b>(b)</b>	$-mg - T = m\ddot{x}$ $-mg - \frac{8mg}{l}(x - \frac{1}{8}l) = m\ddot{x}$ $-\frac{8g}{l}x = \ddot{x}$ <p>SHM, period <math>2\pi\sqrt{\frac{l}{8g}}</math> PRINTED ANSWER</p>	M1 A1 M1dep A1 A1 A1cso (6)
<b>(c)</b>	$a = \frac{1}{2}l - \frac{1}{8}l = \frac{3}{8}l$ $u^2 = \frac{8g}{l}((\frac{3}{8}l)^2 - (\frac{1}{8}l)^2)$ $u = \sqrt{gl}$	B1 M1 A1 A1 (4)
<b>(d)</b>	$x = -a \cos \omega t$ $\dot{x} = a\omega \sin \omega t$ $\sqrt{\frac{9gl}{32}} = \frac{3l}{8} \sqrt{\frac{8g}{l}} \sin \sqrt{\frac{8g}{l}} t$ $\frac{1}{2} = \sin \sqrt{\frac{8g}{l}} t$ $t = \frac{\pi}{6} \sqrt{\frac{l}{8g}}$	M1 A1  M1dep A1 (4)  <b>16</b>

## Notes for Question 7

(a)

M1 for Hooke's law and equating tension to weight

A1cao for  $e = \frac{1}{8}l$ 

(b)

M1 for NL2 vertically, weight and tension needed,  $\ddot{x}$  or  $a$  for acceleration hereA1 for a correct equation with  $\ddot{x}$  or  $a$ M1dep for using HL to replace the tension with an expression in terms of  $x$  Dependent on the previous M mark Must have  $\ddot{x}$  now

A1 for this equation correct

A1 for re-arranging to get  $-\frac{8g}{l}x = \ddot{x}$  oeA1cso for the conclusion SHM and the period  $2\pi\sqrt{\frac{l}{8g}}$  \*

(c)

B1 for using the information in the question to obtain amp  $= \frac{3}{8}l$ M1 for using  $v^2 = \omega^2(a^2 - x^2)$  with *their*  $\omega$  and  $a$ A1 for a correct, unsimplified expression for  $u^2$  in terms of  $l$  and  $g$ A1cao for  $u = \sqrt{gl}$ 

By energy: B1 for EPE, M1 equation, A1 correct equation, A1 answer

(d)

M1 for using  $\dot{x} = a\omega \sin \omega t$  (or  $v$  instead of  $\dot{x}$ ) with *their*  $a$  and  $\omega$  and the given speed

A1 for a fully correct equation

M1dep for solving *their* equation **must use radians**A1cao for  $t = \frac{\pi}{6}\sqrt{\frac{l}{8g}}$  or 0.5235...  $\sqrt{\frac{l}{8g}}$  oe. (if sub for  $g$  seen, must be 2 or 3 sf)

Alternative for (d):

Use  $v^2 = \omega^2(a^2 - x^2)$  with *their*  $\omega$  and  $a$  and the given speed M1 $x = \frac{3l}{16}\sqrt{3}$  or  $x^2 = \frac{27l^2}{256}$  oe A1Use  $x = a \cos \omega t$  with *their*  $x$ ,  $\omega$  and  $a$  and solve in radians M1dep $t = \frac{\pi}{6}\sqrt{\frac{l}{8g}}$  or 0.5235...  $\sqrt{\frac{l}{8g}}$  oe. (if sub for  $g$  seen, must be 2 or 3 sf)

A1cao



Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Mechanics 3R  
(6679/01R)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

#### 'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.

e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

#### 'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

#### 'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

### 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  6. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Mechanics Marking

*(But note that specific mark schemes may sometimes override these general principles)*

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra  $g$  in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of  $g = 9.8$  should be given to 2 or 3 SF.
- Use of  $g = 9.81$  should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

SHM Simple harmonic motion

PCLM Principle of conservation of linear momentum

RHS, LHS Right hand side, left hand side.

Question Number	Scheme	Marks
<b>1 (a)</b>	$\frac{dv}{dx} = 3 \Rightarrow v = 3x - 3$ $a = 3(3x - 3)$ <p>When <math>x = 5</math>, <math>F = 0.25 \times 3(15 - 3) = 9 \text{ N}</math></p>	M1 A1 DM1 A1 (4)
<b>(b)</b>	$\frac{dx}{dt} = 3(x - 1)$ $\int_2^5 \frac{dx}{(x - 1)} = \int_0^t 3 dt$ $[\ln(x - 1)]_2^5 = 3t$ $t = \frac{1}{3} \ln 4 = 0.4620 \dots$	M1 A1 DM1 A1 (4)
<b>Notes</b>		
<b>(a)</b>	M1 Integration A1 correct integration DM1 using $a = v dv/dx$ with their $v$ A1 correct integration	
<b>(b)</b>	M1 using $\frac{dx}{dt} = 3(x - 1)$ A1 correct integrals with correct limits DM1 Substitute the limits A1 correct final answer	

Question Number	Scheme	Marks
<b>2(a)</b>	$T \sin 60^\circ + R \sin 60^\circ = mg$ $T \cos 60^\circ - R \cos 60^\circ = ml \cos 60^\circ \omega^2$ $T = \frac{1}{2} m(l\omega^2 + \frac{2}{\sqrt{3}}g)$	<p>M1 A1</p> <p>M1 A1 A1</p> <p>DM1 A1 (7)</p>
<b>(b)</b>	$R = \frac{1}{2} m(\frac{2}{\sqrt{3}}g - l\omega^2)$ $\frac{1}{2} m(\frac{2}{\sqrt{3}}g - l\omega^2) > 0$ $\omega < \sqrt{\frac{2g}{l\sqrt{3}}}$ $t > 2\pi \sqrt{\frac{l\sqrt{3}}{2g}} \quad **$	<p>M1 A1</p> <p>DM1</p> <p>A1</p> <p>DM1 A1 (6)</p> <p><b>13</b></p>
<b>Notes</b>		
<b>(a)</b>	<p>M1 vertical equation</p> <p>A1 correct vertical equation</p> <p>M1 horizontal equation, acceleration in either form</p> <p>A1 correct lhs</p> <p>A1 correct rhs</p> <p>DM1 solve for <math>T</math></p> <p>A1 correct <math>T</math></p>	
<b>(b)</b>	<p>M1 obtain an expression for <math>R</math></p> <p>A1 correct expression</p> <p>DM1 setting <math>R &gt; 0</math></p> <p>A1 correct inequality for <math>\omega</math></p> <p>DM1 obtaining an inequality for <math>t</math></p> <p>A1 correct inequality</p>	

Question Number	Scheme	Marks
<b>3 (a)</b>	$R = mg \cos \theta$ WD against friction = $\mu x mg \cos \theta$ $\mu x mg \cos \theta = mgx \sin \theta - \frac{mgx^2}{2a}$ $x = 2a(\sin \theta - \mu \cos \theta) \quad **$	B1 B1 M1 A2 A1 (6)
<b>(b)</b>	$T = \frac{mg 2a(\sin \theta - \mu \cos \theta)}{a} = 2mg(\sin \theta - \mu \cos \theta)$ <p>No motion if <math>T \leq mg \sin \theta + \mu mg \cos \theta</math></p> $2mg(\sin \theta - \mu \cos \theta) \leq mg \sin \theta + \mu mg \cos \theta$ $\frac{1}{3} \tan \theta \leq \mu \quad **$	B1 M1 A1 DM1 A1 (5) <b>11</b>
<b>Notes</b>		
<b>(a)</b>	B1 correct equation perpendicular to the plane B1 correct expression for work done against friction M1 work-energy equation A2 fully correct; A1 one error; A1 correct expression for $x$ no errors in the working	
<b>(b)</b>	B1 use Hooke's law to obtain a correct expression for $T$ M1 using NL2 parallel to the plane to set up an inequality for situation where no motion A1 correct inequality DM1 solving to get an inequality for $\mu$ A1 correct inequality and no errors in the working  If only error is use of $<$ instead of $\leq$ , deduct final A mark only	

Question Number	Scheme	Marks
4(a)	$\frac{1}{2}mV^2 - \frac{1}{2}m\frac{2ag}{5} = mga(1 - \cos\theta)$ $mg \cos\theta = m\frac{V^2}{a}$ $V = \sqrt{\frac{4ag}{5}}$	M1 A1 A1 M1 A1 DM1 A1 (7)
(b)	$\cos\theta = \frac{4}{5}$ $t = \frac{a - a \sin\theta}{V \cos\theta} \left( = \sqrt{\frac{5a}{16g}} \right)$ $s = Vt \sin\theta + \frac{1}{2}gt^2$ $= \sqrt{\frac{4ag}{5}} \sqrt{\frac{5a}{16g}} \frac{3}{5} + \frac{1}{2}g\left(\frac{5a}{16g}\right)$ $= \frac{73a}{160}$ $AX = a \cos\theta - \frac{73a}{160}$ $= \frac{11a}{32}$	B1 M1 A1 M1 M1 A1 A1 M1 A1 (9) <b>16</b>
<b>Notes</b>		
(a)	M1 energy equation A1 correct difference of KEs A1 fully correct equation M1 NL2 towards the centre. May include $R$ A1 correct equation May include $R$ DM1 set $R = 0$ and solve for $V$ or $V^2$ A1 correct final answer with no errors in working	
(b)	B1 for correct trig function for $\theta$ M1 using the horizontal distance and speed to obtain an expression for the time A1 correct expression M1 using $s = ut + \frac{1}{2}at^2$ to get the vertical distance M1 attempt at initial vertical velocity A1 correct initial vertical velocity A1 correct vertical distance M1 attempt distance $AX$ A1 correct final answer	



Question Number	Scheme	Marks
<b>5. (a)</b>	$\begin{array}{ccc} \pi r^2 h & \pi(\frac{1}{4}r)^2(\frac{1}{4}h) & \pi r^2 - \pi(\frac{1}{4}r)^2(\frac{1}{4}h) \\ \frac{1}{2}h & \frac{1}{8}h & \bar{y} \end{array}$ $\pi r^2 h \frac{1}{2}h - \pi(\frac{1}{4}r)^2(\frac{1}{4}h)\frac{1}{8}h = \left[ \pi r^2 - \pi(\frac{1}{4}r)^2(\frac{1}{4}h) \right] \bar{y}$ $\bar{y} = \frac{85h}{168} \quad **$	B2 B2  M1 A1ft A1 (7)
<b>(b)</b>	$0 - \pi(\frac{1}{4}r)^2(\frac{1}{4}h)\frac{1}{4}r = \left[ \pi r^2 - \pi(\frac{1}{4}r)^2(\frac{1}{4}h) \right] \bar{x}$ $\bar{x} = -\frac{r}{252}$ $\tan \alpha = \frac{\frac{85h}{168}}{\frac{r}{252}} = 17$ $r = 7.5h$	M1 A1 A1  DM1 A1ft  A1 (6) <b>13</b>
<b>Notes</b>		
<b>(a)</b>	B2 masses or volumes B2 all correct; B1 two correct B2 distances B2 all correct; B1 one of the known ones correct M1A1ft form a moments equation using their volumes and distances A1 correct result with no errors in the working	
<b>(b)</b>	M1A1 form an equation to find the distance of the centre of mass from the axis of the cylinder A1 correct distance DM1 using their two distances to find the tan of the required angle (may be inverted) A1ft ratio is correct (inc correct way up) with their distances A1 correct answer	

Question Number	Scheme	Marks
6(a)	$\frac{4mge}{l} = mg$ $e = \frac{1}{4}l$	M1 A1 (2)
(b)	$mg - T = m\ddot{x}$ $mg - \frac{4mg}{l}\left(x + \frac{1}{4}l\right) = m\ddot{x}$ $-\frac{4g}{l}x = \ddot{x}$ $\text{SHM, } \left( \text{with } \omega = \sqrt{\frac{4g}{l}} \right)$	M1 A1 M1 A1 A1 (5)
(c)	$\sqrt{gl} = a\sqrt{\frac{4g}{l}}$ $a = \frac{1}{2}l$	M1 A1 A1 (3)
(d)	$-\frac{1}{4}l = \frac{1}{2}l \sin\sqrt{\frac{4g}{l}}t$ $t = \frac{7\pi}{12}\sqrt{\frac{l}{g}}$	M1 A1 M1 A1 (4)
<b>14</b>		
Notes		
(a)	M1 using Hooke's law to obtain an equation for $e$ A1 correct answer	
(b)	M1 using NL2 vertically A1 correct equation M1 using Hooke's law to replace $T$ with an expression for $x$ . These 3 marks can be gained with $a$ instead of $\ddot{x}$ A1 fully correct, simplified equation A1 conclusion with all work correct	
(c)	M1 using $v = a\omega$ A1 correct equation A1 correct amplitude	
(d)	M1 for an equation to find required time A1 correct equation M1 solving their equation must be in radians and must give a positive value A1 correct time decimal equivalent acceptable.	





# Mark Scheme (Results)

Summer 2014

Pearson Edexcel International A Level in  
Mechanics 3  
(WME03/01)

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Summer 2014

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL IAL MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

#### 'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.

e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

#### 'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

#### 'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.



### 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  6. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Mechanics Marking

*(But note that specific mark schemes may sometimes override these general principles)*

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra  $g$  in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of  $g = 9.8$  should be given to 2 or 3 SF.
- Use of  $g = 9.81$  should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

SHM Simple harmonic motion

PCLM Principle of conservation of linear momentum

RHS, LHS Right hand side, left hand side.

Question Number	Scheme	Marks
<b>1(a)</b>	$\omega = \frac{2\pi}{\frac{\pi}{4}} = 8$ $\ddot{x} = -\omega^2 x$ $20 =  -8^2 a  = 64a$ $a = \frac{20}{64} = \frac{5}{16} \text{ (m) oe (Accept 0.31, 0.313, 0.3125)}$	M1  M1  A1 (3)
<b>(b)</b>	$v_{\max} = a\omega = 2.5 \text{ (m s}^{-1}\text{)}$	B1ft (1)
<b>(c)</b>	$\text{Dist from A} = 1.5 - \frac{20}{16} = 0.25$ $x = a \cos \omega t = \frac{5}{16} \cos 8t$ $\frac{1}{16} = \frac{5}{16} \cos 8t$ $t = \frac{1}{8} \cos^{-1} 0.2$ $\text{Total time} = \frac{\pi}{4} + \frac{1}{8} \cos^{-1} 0.2 = 0.95657... \text{ 0.96 or better}$	B1   M1  A1  A1ft (4) [8]
<b>Notes for Question 1</b>		
	(a)M1 for attempting to obtain $\omega$ no need to simplify M1 using max mag of $\ddot{x} =  -\omega^2 x $ with max mag accel = 20 and their $\omega$ A1 $a = \frac{5}{16} \text{ m oe fraction or 0.3125 m}$ (b)B1ft $v_{\max} = a\omega = 2.5$ follow through their values for $\omega$ and $a$ (c)B1 finding the distance from A when P has travelled 1.5 m = 0.25 m OR the distance from the centre = $\frac{1}{16}$ M1 using $x = a \cos \omega t$ with $x = \frac{1}{16}$ (their value), their $\omega$ and their $a$ OR for using $x = a \sin \omega t$ with $x = \frac{1}{16}$ (their value), their $\omega$ and their $a$ A1 for $t = \frac{1}{8} \cos^{-1} 0.2$ OR $t = \frac{1}{8} \sin^{-1} 0.2$ A1ft for $\frac{\pi}{4} + \frac{1}{8} \cos^{-1} 0.2 = 0.95657... \text{ 0.96 or better}$ ft their time or equivalent using sine	

Question Number	Scheme	Marks
2.	<p>Mass/area of half of lamina = <math>(\rho) \times \frac{1}{2} \times a \times \sqrt{3}a = (\rho) \frac{\sqrt{3}a^2}{2}</math></p> <p><math>\int_0^{a\sqrt{3}} yx dx = \int_0^{a\sqrt{3}} \frac{x^2}{\sqrt{3}} dx</math></p> <p><math>= \left[ \frac{x^3}{3\sqrt{3}} \right]_0^{a\sqrt{3}}</math></p> <p><math>= a^3</math></p> <p>For the half lamina in the first quadrant <math>\bar{x} = \frac{\int yx dx}{\text{area}} = a^3 \div \frac{a^2\sqrt{3}}{2}</math></p> <p>By symmetry, c of m of complete triangle is <math>\frac{2a}{\sqrt{3}}</math> oe eg 1.15a, 1.2a</p> <p><i>Alternative</i> Work with the whole lamina by multiplying by 2 in lines 1 - 4. No mention of symmetry needed for final answer.</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1 [6]</p>
<b>Notes for Question 2</b>		
	<p>B1 for the mass or area of half of the lamina</p> <p>M1 for attempting to integrate <math>\int_0^{a\sqrt{3}} \frac{x^2}{\sqrt{3}} dx</math> limits not needed here</p> <p>A1 for <math>\left[ \frac{x^3}{3\sqrt{3}} \right]_0^{a\sqrt{3}}</math> limits must be shown and correct but can be implied if result of sub is correct.</p> <p>A1 for sub limits to get <math>a^3</math></p> <p>M1 for using <math>\bar{x} = \frac{\int yx dx}{\text{area}}</math> with their previous answers</p> <p>A1cso for <math>\frac{2a}{\sqrt{3}}</math> oe eg 1.15a, 1.2a</p> <p>"Symmetry" or "2 x " <u>must</u> be seen for all marks to be awarded. If missing, deduct final A mark.</p> <p>If no <math>a</math> in the integrals deduct final A mark unless similar triangles are mentioned.</p> <p>Use of a solid scores 0/6</p>	

Question Number	Scheme	Marks
3	$T_a \cos 30 + T_b \cos 60 = 3g$ $T_a \sin 30 + T_b \sin 60 = 3r\omega^2$ $= 3 \times 0.4 \cos 30 \omega^2$ <p>Solve:</p> $T_a \frac{\sqrt{3}}{2} + \frac{1}{2} T_b = 3g$ $\frac{1}{2} T_a + T_b \frac{\sqrt{3}}{2} = 3 \times 0.4 \times \frac{\sqrt{3}}{2} \times 36$ $T_b = 1.2 \times 36 \times \frac{3}{2} - 3g$ $T_b = 35.4 \text{ (N)}$ $T_a = 13.5 \text{ (N) must be 2 or 3 sf}$	<p>M1A1A1</p> <p>M1A1</p> <p>A1</p> <p>DM1A1</p> <p>A1 [9]</p>
	<b>Notes for Question 3</b>	
	<p>M1 for resolving vertically. Two tensions (resolved) and a weight must be seen.</p> <p>A1 for two correct terms</p> <p>A1 for all terms (inc signs) correct</p> <p>M1 for NL2 horizontally. Two tensions (resolved) and mass x acceleration needed. The acceleration can be in either form</p> <p>A1 for the two tensions, correctly resolved and added</p> <p>A1 for <math>3 \times 0.4 \cos 30 \omega^2</math></p> <p>M1 dep for solving the equations to obtain either tension. Dependent on both previous M marks</p> <p>A1 for either tension correct</p> <p>A1 for the second tension correct. Both tensions must be given to 2 or 3 sf to gain the marks. (Penalise once for more than 3 sf)</p>	

Question Number	Scheme	Marks
4(a)	$0.4 \frac{dv}{dt} = \frac{4}{(t+5)^2}$ $v = -\frac{10}{(t+5)} + c$ $t = 0, v = 4 \Rightarrow 4 = -\frac{10}{5} + c, c = 6$ $v = 6 - \frac{10}{(t+5)} \quad t \geq 0 \quad \frac{10}{t+5} \geq 0 \Rightarrow v \leq 6$	B1 M1A1 DM1 A1 (5)
(b)	$s = \int_2^7 \left( 6 - \frac{10}{(t+5)} \right) dt$ $= \left[ 6t - 10 \ln(t+5) \right]_2^7$ $= 42 - 10 \ln 12 - (12 - 10 \ln 7)$ $= 30 + 10 \ln \left( \frac{7}{12} \right) \quad \text{oe eg } 24.6100 \dots 25 \text{ or better}$	M1A1ft M1 A1 (4)
(c)	$\text{KE} = \frac{1}{2} \times 0.4 \times \left( 6 - \frac{10}{12} \right)^2 - \frac{1}{2} \times 0.4 \times \left( 6 - \frac{10}{7} \right)^2$ $= 1.1592 \dots \text{J} \quad \text{Accept } 1.2 \text{ or better}$	M1A1ft A1 (3) [12]
<b>Notes for Question 4</b>		
	<p>(a) B1 for a correct equation of motion with acceleration = <math>\frac{dv}{dt}</math>. Can be awarded by implication if work correct at next stage</p> <p>M1 for attempting the integration wrt <math>t</math> to obtain an expression for <math>v</math></p> <p>A1 for correct result, constant not needed</p> <p>M1dep for using <math>t = 0, v = 4</math> to obtain a value for <math>c</math> Dependent on the previous M mark</p> <p>A1cso for a correct concluding statement. Can have <math>\geq</math> or <math>&gt;</math></p> <p>(b) M1 for attempting the integration of <i>their</i> expression for <math>v</math> Limits need not be seen for this mark</p> <p>A1ft for correct integration</p> <p>M1 for substituting the limits 2 and 7</p> <p>A1cao a correct result, exact or decimal (min 2 sf)</p> <p>(c) M1 for attempting the difference of KE between the points A and B (either way round). Velocities to be calculated using <i>their</i> expression for <math>v</math>. Award for a gain or a loss.</p> <p>A1ft for KE at B - KE at A, with <i>their</i> expression for <math>v</math>. Need not be simplified, may be reversed.</p> <p>A1cso for = 1.1592...J Accept 1.2 or better Must be positive.</p>	

Question Number	Scheme	Marks
5(a)	Energy A to B $\frac{1}{2} \times 2mv^2 - \frac{1}{2} \times 2mu^2 = 2mga(1 - \cos 60^\circ)$ $v^2 = u^2 + ga$ C of M: $2mv = 3mV$ $V = \frac{2}{3}\sqrt{u^2 + ag}$ *	M1A1 A1  B1 DM1A1 (6)
(b)	NL2 at bottom: $3m \frac{V^2}{a} = T - 3mg$ $T = 3m \left( \frac{V^2}{a} + g \right) = m \left( \frac{4u^2}{3a} + \frac{13g}{3} \right)$ (N) oe	M1A1  A1 (3)
(c)	Energy from B to top: $\frac{1}{2} \times 3m \times \frac{4}{9}(u^2 + ag) - \frac{1}{2} \times 3mX^2 = 3mg \times 2a$ At top $T + 3mg = 3m \frac{X^2}{a}$ $T \geq 0 \Rightarrow X^2 \geq ag$ $\frac{4}{18}(u^2 + ag) - 2ag \geq \frac{ag}{2}$ $u^2 \geq \frac{41ag}{4}$ *	M1A1  M1A1 DM1  A1 (6) [15]

	Notes for Question 5	
	<p>(a)</p> <p>M1 for an energy equation from A to B. Two KE terms and 2 PE terms (or a loss of PE) needed.</p> <p>A1 for correct KE terms (difference either way round)</p> <p>A1 for a correct loss of PE and all signs correct throughout the equation mass can be <math>m</math> or <math>2m</math> for these two A marks, provided consistent</p> <p>B1 for a correct conservation of momentum equation</p> <p>M1dep for using the two equations to obtain the speed of the combined particle. Dep on the first M mark and using the C of M equation even if B0 has been given for it.</p> <p>A1cso for <math>V = \frac{2}{3}\sqrt{u^2 + ag}</math> *</p> <p>(b)</p> <p>M1 for using NL2 at the bottom, tension, weight and mass x accel terms required. Accel can be in either form.</p> <p>A1 for a fully correct equation, no need to substitute for the speed.</p> <p>A1 for substituting the speed (as given in (a)) to obtain a correct expression for the tension in terms of <math>a</math>, <math>g</math>, <math>m</math> and <math>u</math>. Must be simplified.</p> <p>Any equivalent expression scores A1 eg <math>\frac{m}{3a}(12u^2 + 13ag)</math></p> <p>(c)M1 An energy equation from the bottom to the top. Must have a difference of KE terms and a gain of PE.</p> <p>A1 for a fully correct equation</p> <p>M1 for NL2 along the radius at the top. Must have a tension, weight and mass x acceleration (in either form).</p> <p>A1 for a fully correct equation acceleration in either form.</p> <p>M1dep for using <math>T \geq 0</math> at the top to obtain an inequality for the speed at the top and completing to an inequality for <math>u^2</math>. Dependent on both previous M marks in (c). OR: Eliminate <math>X^2</math> between the two equations and then use the inequality <math>T \geq 0</math></p> <p>A1cso for <math>u^2 \geq \frac{41ag}{4}</math> *</p>	



Question Number	Scheme	Marks
6(a)	$T = \frac{9mgpa}{6a} = mg$ $p = \frac{2}{3} *$	M1 A1 (2)
(b)	$T = \frac{9mg\left(\frac{2}{3}a + x\right)}{6a}$ $mg - \frac{9mg\left(\frac{2}{3}a + x\right)}{6a} = m\ddot{x}$ $-\frac{9gx}{6a} = -\frac{3gx}{2a} = \ddot{x}$ <p>Of form <math>\ddot{x} = -\omega^2 x \therefore</math> SHM</p>	M1A1 DM1 A1 (4)
(c)	$\text{Period} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{3g}{2a}}}, = 2\pi\sqrt{\frac{2a}{3g}}$	M1,A1ft (2)
(d)	The string never becomes slack or the SHM is complete	B1 (1)
(e)	$\text{Loss of EPE} = \frac{9mg \times (2a)^2}{2 \times 6a} = 3mga$ $mgh = 3mga, \quad h = 3a$ $AE = AD - h = 8a - 3a = 5a$	B1 M1,A1 A1ft (4) [13]
<b>Notes for Question 6</b>		
	<p>(a)M1 for using Hooke's Law resolving vertically. A1cso for <math>p = \frac{2}{3} *</math></p> <p>(b)M1 for an equation of motion vertically. Must have a tension, a weight and a mass x acceleration. Allow with <math>a</math> for acceleration. Must be dimensionally correct, but allow for misuse of brackets. A1 for a correct equation, can still have <math>a</math> M1dep for rearranging to the form <math>\ddot{x} = -\omega^2 x</math> Acceleration <math>a</math> scores M0 A1 for a correct equation and a conclusion eg <math>\therefore</math> SHM Accept "shown"</p> <p>(c)M1 for using period <math>= \frac{2\pi}{\omega}</math> with <i>their</i> <math>\omega</math> to obtain the period. A1ft for <math>2\pi\sqrt{\frac{2a}{3g}}</math></p> <p>(d)B1 for any statement equivalent to those shown</p> <p>(e)B1 for the EPE lost or initial EPE. Need not be simplified. M1 for an energy equation equating their EPE to the PE gained A1 for a correct vertical distance risen A1ft for <math>AE = 8a - \text{their distance risen}</math></p>	

Question Number	Scheme	Marks
7(a)	<p>Cylinder                      cone                      <math>S</math></p> <p>Mass                      <math>18\pi a^3</math>                      <math>\frac{1}{3}\pi \times 4 \times \frac{3}{2}a^3 = 2\pi a^3</math>                      <math>16\pi a^3</math></p> <p>Dist from <math>O</math>                      <math>a</math>                      <math>\frac{3a}{8}</math>                      <math>\bar{x}</math></p> <p><math>18a - 2 \times \frac{3}{8}a = 16\bar{x}</math></p> <p><math>\bar{x} = \frac{69}{64}a</math> *</p>	<p>B1</p> <p>B1</p> <p>M1A1ft</p> <p>A1 (5)</p>
(b)	<p><math>\tan \theta = \frac{3a}{\bar{x}}</math></p> <p><math>\theta = \tan^{-1}\left(\frac{64}{23}\right), \quad \theta = 70.23\dots^\circ = 70^\circ \text{ or } 1.2257\dots\text{rad}</math></p>	<p>M1</p> <p>A1,A1 (3)</p>
(c)	<p><math>R = mg \cos \phi \quad F = mg \sin \phi</math> (or <math>F \cos \phi = R \sin \phi</math> M1A1)</p> <p><math>\frac{F}{R} = \tan \phi = \mu = 0.6</math></p> <p><math>\phi_{\max} = 30.96\dots = 31</math> *</p>	<p>B1 B1</p> <p>M1</p> <p>A1 (4)</p> <p>[12]</p>
Notes for Question 7		
	<p>(a)</p> <p>B1 for a correct mass ratio</p> <p>B1 for correct distances, probably from <math>O</math> but can be from another point, eg vertex of cone or centre of base.</p> <p>M1 for a moments equation with 3 terms</p> <p>A1ft for a correct equation, follow through <i>their</i> mass ratio and distances.</p> <p>A1cso for <math>\bar{x} = \frac{69}{64}a</math> *</p> <p>(b)</p> <p>M1 for <math>\tan \theta = \frac{3a}{\bar{x}}</math> with the given <math>\bar{x}</math>. Can be either way up.</p> <p>A1 for <math>\theta = \tan^{-1}\left(\frac{64}{23}\right)</math></p> <p>A1cao for <math>\theta = 70.23\dots^\circ</math> Accept <math>70^\circ</math> or better. Or <math>1.2257\dots^\circ</math> Accept <math>1.2^\circ</math> or better.</p> <p>(c)</p> <p>B1 for <math>R = mg \cos \phi</math></p> <p>B1 for <math>F = mg \sin \phi</math></p> <p>M1 for using <math>F = \mu R</math> with <math>\mu = 0.6</math> to obtain an equation for <math>\tan \phi</math></p> <p>A1cso for <math>\phi_{\max} = 31^\circ</math> *</p>	





# Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Mechanics 3  
(6679\_01)

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Summer 2014

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**PEARSON EDEXCEL GCE MATHEMATICS****General Instructions for Marking**

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.

e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.



### 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  6. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Mechanics Marking

*(But note that specific mark schemes may sometimes override these general principles)*

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra  $g$  in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- dM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of  $g = 9.8$  should be given to 2 or 3 SF.
- Use of  $g = 9.81$  should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

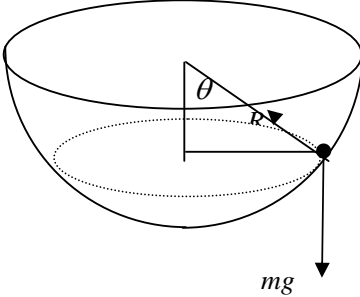
NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

SHM Simple harmonic motion

PCLM Principle of conservation of linear momentum

RHS, LHS Right hand side, left hand side.

Question Number	Scheme	Marks
1.	 $R \sin \theta = m \times 4r \sin \theta \times \frac{3g}{8r}$ $R = \frac{3}{2}mg$ $R \cos \theta = mg$ $\frac{3}{2}mg \cos \theta = mg$ $\cos \theta = \frac{2}{3}$ $OC = 4r \cos \theta = 4r \times \frac{2}{3} = \frac{8}{3}r \text{ oe}$	<p>M1A1A1</p> <p>M1A1</p> <p>M1(dep)</p> <p>A1</p> <p>M1A1</p> <p>[9]</p>
<b>Notes for Question 1</b>		
M1	for NL2 towards C - Accept use of $v = \sqrt{\frac{3g}{8r}}$ and $a = \frac{v^2}{r}$ as a mis-read	
A1	for LHS fully correct	
A1	for RHS fully correct	
<b>ALT:</b>	Work in the direction of R and obtain the same equation with $\sin \theta$ "cancelled". Give M1A1A1 if fully correct, M0 otherwise.	
M1	for resolving vertically	
A1	for the equation fully correct	
M1 dep	for eliminating R between the two equations Dependent on both above M marks	
A1	for $\cos \theta = \frac{2}{3}$	
M1	for attempting to use trig or Pythagoras to obtain OC	
A1 cso	for $OC = \frac{8}{3}r$	

Alternative for Question 1	
M1A1A1	$R \sin \theta = m \times a \times \frac{3g}{8r}$
M1 A1	$R \cos \theta = mg$
M1 A1	$\tan \theta = \frac{3a}{8r}$
M1	$\frac{a}{OC} = \frac{3a}{8r}$
A1	$OC = \frac{8r}{3}$



Notes for Question 2	
(a)	for $\frac{k}{R^2} = mg$ . If not made clear that this applies at the surface of the Earth award M0 <b>or</b>
M1	$\frac{k}{x^2} = mg$ <b>and</b> $x = R$ .
A1 cso	for $k = mgR^2$ *
(b)	
M1	for using accel = $v \frac{dv}{dx}$ oe in NL2 with or w/o $m$ Minus sign not required.
M1 dep	for attempting to integrate both sides - minus not needed
A1	for fully correct integration, with or w/o the constant. Must have included the minus sign from the start.
M1 dep	for using $x = \frac{5R}{4}$ , $v = \sqrt{\frac{gR}{2}}$ to obtain a value for the constant. Use of $x = \frac{R}{4}$ scores M0 Depends on both previous M marks
A1	for $c = -\frac{11gR}{20}$
M1 dep	for setting $v = 0$ and solving for $x$ Depends on 1st and 2nd M marks, but not 3rd
A1 cso	for $x = \frac{20R}{11}$
<b>ALT:</b>	By definite integration First 3 marks as above, then
DM1	Using limits $x = \frac{5R}{4}$ , $v = \sqrt{\frac{gR}{2}}$
DM1	Using limit $v = 0$
A1	Correct substitution
A1 cso	for $x = \frac{20R}{11}$
<b>NB:</b> The penultimate A mark has changed position, but must be entered on e-pen in its original position.	

## Alternative for Question 2

**Qu 2 (a):**

Using  $F = \frac{GM_1M_2}{x^2}$  with  $x = R$  and one mass as mass of Earth:

$$mg = \frac{GmM_E}{R^2}$$

$$GM_E = gR^2 \Rightarrow F = \frac{mgR^2}{x^2} \Rightarrow F = \frac{k}{x^2} \text{ with } k = mgR^2 *$$

M1 Complete method A1 Correct answer

**Qu 2 (b):**

By conservation of energy:

$$\text{Work done against gravity} = \int_{\frac{5r}{4}}^z \frac{mgR^2}{x^2} dx = \int_{\frac{5r}{4}}^z mgR^2 x^{-2} dx$$

M1

$$= \frac{4mgR}{5} - \frac{mgR^2}{z}$$

DM1(integration)A1(correct)

$$\text{Work-energy equation: } \frac{mgR}{4} = \frac{4mgR}{5} - \frac{mgR^2}{z}$$

DM1A1

$$z = \frac{20R}{11}$$

DM1A1

Question Number	Scheme	Marks												
<b>3.</b> <b>(a)</b>	<table> <tr> <td>Shell</td><td>wax</td><td>filled shell</td></tr> <tr> <td><math>m</math></td><td><math>3m</math></td><td><math>4m</math></td></tr> <tr> <td>Dist. above vertex</td><td><math>\frac{2}{3} \times 6r</math></td><td><math>\frac{3}{4} \times 2r</math></td></tr> <tr> <td></td><td></td><td><math>\bar{x}</math></td></tr> </table> $4mr + \frac{9}{2}mr = 4m\bar{x}$ $\bar{x} = \frac{17}{8}r$	Shell	wax	filled shell	$m$	$3m$	$4m$	Dist. above vertex	$\frac{2}{3} \times 6r$	$\frac{3}{4} \times 2r$			$\bar{x}$	B1  M1A1ft  A1 (4)
Shell	wax	filled shell												
$m$	$3m$	$4m$												
Dist. above vertex	$\frac{2}{3} \times 6r$	$\frac{3}{4} \times 2r$												
		$\bar{x}$												
<b>(b)</b>	$\tan \theta = \frac{r}{6r - \bar{x}} = \frac{r}{31r/8}$ $\tan \theta = \frac{8}{31}$ $\theta = 14.47...^\circ$	M1A1ft   A1 (3) <b>[7]</b>												
<b>Notes for Question 3</b>														
<b>(a)</b> B1 M1 A1 ft A1 cso	for correct distances from the vertex or any other point for a dimensionally correct moments equation with their distances and masses for a correct moments equation, follow through their distances but must have correct masses for $\bar{x} = \frac{17}{8}r$  <b>NB:</b> If $\frac{2}{3}$ and $\frac{3}{4}$ are interchanged in the distances, the correct answer is obtained but the solution is incorrect. Score: B0M1A1A0													
<b>(b)</b> M1 A1 ft A1 cso	for $\tan \theta = \frac{r}{6r - \bar{x}}$ . Can be either way up, but must include $6r - \bar{x}$ . Substitution for $\bar{x}$ not required for $\tan \theta = \frac{r}{31r/8}$ oe ft their $\bar{x}$ for $\theta = 14.47...^\circ$ Accept $14^\circ$ , $14.5^\circ$ or better or $\theta = 0.2525... \text{rad}$ Accept 0.25 or better Obtuse angle accepted.													



Question Number	Scheme	Marks
<b>4</b>		
<b>(a)</b>	$\frac{3mgx^2}{2l} = 2mgx \sin \alpha$ $3x^2 = 4xl \times \frac{3}{5}$ $5x^2 = 4xl$ $x = \frac{4}{5}l$	M1A1 B1(A1 on e-pen)
<b>(b)</b>	$R = 2mg \cos \alpha \left( = \frac{8}{5}mg \right)$ $\frac{3mg}{2l} \times \frac{4}{25}l^2 = 2mg \times \frac{2}{5}l \times \frac{3}{5} - , \quad \mu \frac{8}{5}mg \times \frac{2}{5}l$ $6 = 12 - 16\mu$ $16\mu = 6 \quad \mu = \frac{3}{8}$	DM1A1 (5)  B1  M1A1ft, B1ft (A1 on e-pen)  DM1A1 (6) <b>[11]</b>

Notes for Question 4	
(a)	
M1	for an energy equation with an EPE term of the form $\frac{kmgx^2}{l}$ and a GPE term. If a KE term is included it must become 0 later.
A1	for a correct EPE term
B1	for a correct GPE term. This can be in terms of the distance moved down the plane or the vertical distance fallen
M1 dep	for solving their equation to obtain the distance moved or using the vertical distance and obtaining the distance moved along the plane.
A1	for $x = \frac{4}{5}l$ oe eg $x = \frac{12}{15}l$
(b)	
B1	for resolving perpendicular to the plane to obtain $R = 2mg \cos \alpha$ . May only be seen in an equation.
M1	for an work-energy equation with an EPE term of the form $\frac{kmgx^2}{l}$ , a GPE term and the work done against friction. The work term must include a distance along the plane.
A1	for EPE and GPE terms correct and work subtracted from the GPE
B1 ft	for the work term ft their $R$
M1 dep	for solving to obtain a value for $\mu$
A1 cso	for $\mu = \frac{3}{8}$ oe inc 0.375 but not 0.38
<b>If <math>m</math> used instead of <math>2m</math>, assuming correct otherwise:</b>	
(a)	M1A1B0M1A0 (so 2 penalties for mis-read)
(b)	
B1	$R = mg \cos \alpha$
M1, A1	Equation, with EPE correct and $mg \times \frac{2}{5}l \times \frac{3}{5}$
B1 ft	$\mu \frac{4mg}{5} \times \frac{2}{5}l$
DM1, A1	$\mu = 0$

## Alternative for Question 4

Qu 4: Using NL2:

(a)

$$2ma = 2mg \sin \alpha - \frac{3mgx}{l}$$

$$2v \frac{dv}{dx} = \frac{6g}{5} - \frac{3gx}{l}$$

$$v^2 = \frac{6gx}{5} - \frac{3gx^2}{2l}, +c$$

$$v = 0 \quad 3gx \left( \frac{2}{5} - \frac{x}{2l} \right) = 0$$

$$x = \frac{4l}{5}$$

M1(equation and attempt integration)

A1, A1 (show  $c = 0$ )M1 (set  $v = 0$  and solve)

A1

(b)

$$R = 2mg \cos \alpha$$

$$2v \frac{dv}{dx} = \frac{6g}{5} - \frac{3gx}{l} - \mu \frac{8g}{5}$$

$$v^2 = \frac{6gx}{5} - \frac{3gx^2}{2l} - \mu \frac{8gx}{5}, +c$$

$$v = 0 \quad x = \frac{2l}{5} \quad \mu \frac{8}{5} = \frac{6}{5} - \frac{3}{2l} \times \frac{2l}{5}$$

$$\mu = \frac{3}{8}$$

B1

M1(eqn and int)A1, A1 (show  $c = 0$ )M1 (set  $v = 0$  and solve)

A1

If SHM methods are used, SHM must be proved first.

Question Number	Scheme	Marks
<p><b>5.</b></p> <p><b>(a)</b></p>	$\text{Vol} = \pi \int_0^{\frac{\pi}{2}} y^2 dx = \pi \int_0^{\frac{\pi}{2}} \cos^2 x dx$ $= \pi \int_0^{\frac{\pi}{2}} \frac{1}{2} (\cos 2x + 1) dx$ $= \frac{\pi}{2} \left[ \frac{1}{2} \sin 2x + x \right]_0^{\frac{\pi}{2}} = \frac{\pi^2}{4}$ <p><b>(b)</b></p> $\pi \int_0^{\frac{\pi}{2}} y^2 x dx = \pi \int_0^{\frac{\pi}{2}} x \cos^2 x dx$ $= \pi \int_0^{\frac{\pi}{2}} \frac{1}{2} x (\cos 2x + 1) dx$ $= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} x \cos 2x dx + \frac{\pi}{2} \left[ \frac{x^2}{2} \right]_0^{\frac{\pi}{2}}$ $\frac{\pi}{2} \left[ x \times \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{2}} - \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{1}{2} \sin 2x dx, + \frac{\pi^3}{16}$ $= 0 + \frac{\pi}{2} \left[ \frac{1}{4} \cos 2x \right]_0^{\frac{\pi}{2}} + \frac{\pi^3}{16}$ $= \frac{\pi}{8} [-1 - 1] + \frac{\pi^3}{16} = \frac{\pi^3}{16} - \frac{\pi}{4}$ $\bar{x} = \frac{\pi^3 - 4\pi}{16} \div \frac{\pi^2}{4} = \frac{\pi^2 - 4}{4\pi} \text{ or } 0.467088....$	<p>M1</p> <p>M1</p> <p>DM1A1 (4)</p> <p>M1</p> <p>M1,B1</p> <p>DM1</p> <p>A1ft</p> <p>M1A1 (7)</p> <p><b>[11]</b></p>

Notes for Question 5	
(a)	
M1	for using $\text{Vol} = \pi \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ . If $\pi$ is missing here it must be included later to earn this mark.
	Limits not needed
M1	for using the double angle formula (correct) to prepare for integration. Formula must be correct.
	$\pi$ and limits not needed for this mark.
M1 dep	for attempting to integrate and substitute the correct limits (only sub of non-zero limit needed be to seen) dependent on both M marks.
A1 cso	for $\frac{\pi^2}{4}$ * (check integration is correct, answer can be obtained by luck due to the limits)
(b)	<b>NB:</b> The first 5 marks can be earned with or without $\pi$
M1	for using $\pi \int_0^{\frac{\pi}{2}} x \cos^2 x \, dx$ $\pi$ not needed; limits not needed.
M1	for using the double angle formula (correct) and attempting the first stage of integration by parts
B1	for $\frac{\pi^3}{16}$ or $\frac{\pi^2}{16}$ if $\pi$ not included. NB integration by parts not needed for this mark
M1 dep	for completing the integration by parts, limits not needed yet
A1 ft	for $= \frac{\pi}{8}[-1-1] + \frac{\pi^3}{16} = \frac{\pi^3}{16} - \frac{\pi}{4}$ or $= \frac{1}{8}[-1-1] + \frac{\pi^2}{16} = \frac{\pi^2}{16} - \frac{1}{4}$ ft on $\frac{\pi^3}{16}$
M1	for using $\bar{x} = \frac{\int \pi y^2 x \, dx}{\int \pi y^2 \, dx}$ The numerator integral need not be correct.
	$\pi$ should be seen in both or neither integral
	for $\bar{x} = \frac{\pi^2 - 4}{4\pi}$ oe eg $\frac{\pi}{4} - \frac{1}{\pi}$ or 0.467088....
A1 cso	Accept 0.47 or better but no fractions within fractions
	(a) has a given answer, so the cso applies to the solution of (b) only.

Question Number	Scheme	Marks
6.	<div data-bbox="581 247 764 453" style="text-align: center;"> </div> <p data-bbox="224 541 256 573">(a)</p> $\frac{1}{2}mU^2 - \frac{1}{2}mv^2 = 2mga$ $T + mg = m\frac{v^2}{a}$ $T = \frac{(mU^2 - 4mga)}{a} - mg$ $T = \frac{mU^2 - 5mga}{a}$ $T \geq 0 \Rightarrow U^2 \geq 5ga$ $U \geq \sqrt{5ag} \quad *$ <p data-bbox="224 1182 256 1213">(b)</p> <p data-bbox="321 1161 727 1234">At top: <math>T = \frac{9mga - 5mga}{a} = 4mg</math></p> <p data-bbox="321 1276 667 1350">At bottom: <math>T' - mg = \frac{mU^2}{a}</math></p> $kT = mg + \frac{9mag}{a} = 10mg$ $k = \frac{10mg}{4mg} = \frac{5}{2}$	<p data-bbox="1255 548 1328 573">M1A1</p> <p data-bbox="1255 657 1328 682">M1A1</p> <p data-bbox="1255 779 1312 804">DM1</p> <p data-bbox="1255 898 1287 924">A1</p> <p data-bbox="1255 997 1312 1022">DM1</p> <p data-bbox="1255 1077 1409 1108">A1 (8)</p> <p data-bbox="1255 1161 1377 1224">M1(either tension)A1</p> <p data-bbox="1255 1297 1287 1323">A1</p> <p data-bbox="1255 1409 1312 1434">DM1</p> <p data-bbox="1255 1518 1409 1549">A1 (5)</p> <p data-bbox="1255 1577 1304 1608"><b>[13]</b></p>

Notes for Question 6	
(a)	for an energy equation, from the bottom to the top. A difference of KE terms and a PE term needed.
M1	From bottom to a general point gets M0 until a value for $\theta$ at the top is used. $v^2 = u^2 + 2as$ scores M0
A1	for all terms correct (inc signs)
M1	for NL2 along the radius at the top. Two forces and mass x acceleration needed. Accel can be in either form here. <b>But see NB at end of (a)</b>
A1	for a fully correct equation. Acceleration should be $\frac{v^2}{a}$ now.
M1 dep	for eliminating $v$ (vel at top) between the two equations. Dependent on both previous M marks. If $v$ is set = 0, award M0
A1	for a correct expression for $T$
M1 dep	for using $T \geq 0$ to obtain an inequality for $U^2$ or $U$ . Allow with $>$ Dependent on all previous M marks.
A1 cso	for $U \geq \sqrt{5ag}$ * Watch square root! Give A0 if $>$ seen on previous line.
	<b>NB:</b> The second and fourth M marks (and their As if earned) can be given together if $mg \leq m \frac{v^2}{a}$ is seen
(b)	
M1	for obtaining an expression for the tension at the top or at the bottom, no need to substitute for $U$ yet.
A1	Substitute for $U$ and obtain one correct tension ( $4mg$ at top or $10mg$ at bottom)
A1	for the other tension correct
M1 dep	for using tension at bottom = $k$ x tension at the top and solving for $k$
A1 cso	for $k = \frac{5}{2}$ oe

Question Number	Scheme	Marks
7.		
(a)	$T = \frac{\lambda x}{l} = \frac{\lambda \times 0.5l}{l}$ $\lambda = 2mg *$	M1A1 A1 (3)
(b)	$mg - T = m\ddot{x}$ $mg - \frac{2mg(0.5l + x)}{l} = m\ddot{x}$ $\ddot{x} = -\frac{2gx}{l}$ <p><math>\therefore</math> SHM</p>	M1 DM1A1A1 A1 A1cso(B1 on e-pen) (6)
(c)	$a = 0.3l$ $ \ddot{x} _{\max} = 2g \times \frac{0.3l}{l} = 0.6g \quad (= 5.88 \text{ or } 5.9 \text{ m s}^{-2})$	M1A1ft (2)
(d)	$x = a \cos \omega t = 0.3l \cos \left( \sqrt{\frac{2g}{l}} t \right)$ <p>Time C to D: <math>0.15 = 0.3 \cos \left( \sqrt{\frac{2g}{l}} t \right)</math></p> $t = \sqrt{\frac{l}{2g}} \cos^{-1} 0.5$ <p>Time C to E: <math>t' = \text{half period} = \pi \sqrt{\frac{l}{2g}}</math></p> $\text{Time D to E: } = (\pi - \cos^{-1} 0.5) \sqrt{\frac{l}{2g}} = \frac{2\pi}{3} \sqrt{\frac{l}{2g}}$	M1 B1 M1A1 (4)
		<b>[15]</b>

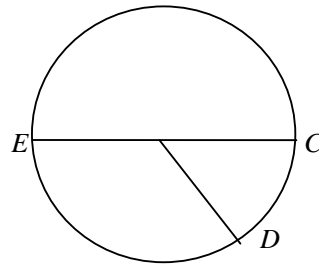


Notes for Question 7	
(a)	
M1	for using Hooke's Law
A1	for a correct equation
A1	for solving to get $\lambda = 2mg$ *
(b)	
M1	for using NL2. Weight and tension must be seen. Acceleration can be $a$ here, but must be an equation at a general position
M1 dep	for using Hooke's Law for the tension. Acceleration can be $a$
A1 A1	for a fully correct equation inc acceleration as $\ddot{x}$ (-1 ee)
A1	for simplifying to $\ddot{x} = -\frac{2gx}{l}$ oe
A1 cso	for the conclusion
(c)	
M1	for using $ \ddot{x} _{\max} = \omega^2 a$ with their $\omega$ and $a = 0.3l$ . $\omega$ must be dimensionally correct
A1 ft	for obtaining the max magnitude of the accel, accept 0.6g, 5.9 or 5.88 only. ft their $\omega$
(d)	
M1	for using $x = a \cos \omega t$ with $x = \pm 0.15l$ , $a = 0.3l$ and their $\omega$ to obtain an expression for the time from $C$ to $D$
B1	for time $C$ to $E$ = half period = $\pi \sqrt{\frac{l}{2g}}$
M1	For any correct method for obtaining the time from $D$ to $E$
A1 cao	for $\frac{2\pi}{3} \sqrt{\frac{l}{2g}}$ oe inc $0.473\sqrt{l}$ $0.47\sqrt{l}$
<b>ALT for</b>	
<b>(d):</b>	
(i)	
M1	Use $x = a \sin \omega t$ with $x = 0.15l$ , $a = 0.3l$ and their $\omega$ to obtain an expression for the time from $B$ to $D$
M1, A1	as above
(ii)	
	Using $x = a \cos \omega t$ with $x = \pm 0.15l$ , $a = 0.3l$ and their $\omega$
	This gives the required time in one step.
	Award M2 A1 for correct substitution
	A1 correct answer
	<b>However do not isw if further work shown. Mark according to mark scheme method and give max M1B1M0A0.</b>

## Alternative for Question 7

Qu 7 (d)

By reference circle:

Centre of circle is  $O$ Angle  $COD = \theta$     Angle  $EOD = \alpha$ 

$$\cos \theta = \frac{0.15l}{0.3l} \quad \theta = \frac{\pi}{3}$$

M1

$$\alpha = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

B1

$$\omega = \sqrt{\frac{2g}{l}}$$

$$\text{time} = \frac{\alpha}{\omega} = \frac{2\pi/3}{\sqrt{\frac{2g}{l}}} = \frac{2\pi}{3} \sqrt{\frac{l}{2g}}$$

M1A1





# Mark Scheme (Results)

January 2015

Pearson Edexcel International A Level in  
Mechanics 3  
(WME03/01)

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January 2015

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL IAL MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

#### 'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.

e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

#### 'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

#### 'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

### 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol  $\checkmark$  will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working



- awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - □ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. If a candidate makes more than one attempt at any question:
- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
6. Ignore wrong working or incorrect statements following a correct answer.

### General Principles for Mechanics Marking

*(But note that specific mark schemes may sometimes override these general principles)*

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra  $g$  in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of  $g = 9.8$  should be given to 2 or 3 SF.
- Use of  $g = 9.81$  should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

SHM Simple harmonic motion

PCLM Principle of conservation of linear momentum

RHS, LHS Right hand side, left hand side.

Jan 2015  
WME03/01 M3 (IAL)  
Mark Scheme

Question Number	Scheme	Marks
1.	$3v \frac{dv}{dx} = \frac{9}{2}(26 - x)$ $\frac{d\left(\frac{1}{2}v^2\right)}{dx} = \frac{3}{2}(26 - x)$ $\frac{1}{2}v^2 = \frac{3}{2}\left(26x - \frac{1}{2}x^2\right) \quad (+c)$ <p>Max speed when accel is zero ie when <math>x = 26</math></p> $\frac{1}{2} \times 32^2 = \frac{3}{2} \times \frac{1}{2} \times 26^2 + c \quad \Rightarrow c = 5$ $v^2 = 3\left(26x - \frac{1}{2}x^2\right) + 10$	<p>M1</p> <p>M1A1</p> <p>B1</p> <p>A1</p> <p>A1 (6)</p>
	<p>M1 NL2 with accel in a correct form- can be implied by subsequent working</p> <p>M1 integrate the equation wrt <math>x</math></p> <p>A1 correct result after integrating - constant not needed</p> <p>B1 deduce max speed occurs when <math>x = 26</math></p> <p>A1 a correct value for the constant</p> <p>A1 a correct expression for <math>v^2</math> - can be in any form</p> <p>ALT for last 3 marks:</p> <p>M1 (B1 on e-pen) Complete square and equate constant part to <math>32^2</math> or use</p> $\text{max of quadratic} = \frac{4ac - b^2}{4a}$ <p>A1 correct <math>c</math></p> <p>A1 correct expression for <math>v^2</math></p>	

Question Number	Scheme	Marks
<p><b>2</b></p> <p>(i)</p> <p>(ii)</p>	$\text{Area} = \int_1^3 y \, dx = \int_1^3 \frac{3}{x^2} \, dx$ $= \left[ -3x^{-1} \right]_1^3 = -1 - (-3) = 2$ $\int_1^3 xy \, dx = \int_1^3 x \times \frac{3}{x^2} \, dx = \int_1^3 \frac{3}{x} \, dx$ $\left[ 3 \ln x \right]_1^3 \quad (= 3 \ln 3)$ $\bar{x} = \frac{3 \ln 3}{2} \quad (= 1.647 \dots)$ $\int_1^3 \frac{1}{2} y^2 \, dx = \int_1^3 \frac{1}{2} \times \frac{9}{x^4} \, dx$ $\frac{9}{2} \left[ -\frac{x^{-3}}{3} \right]_1^3 = \frac{9}{2} \left[ -\frac{1}{81} + \frac{1}{3} \right] = 1\frac{4}{9}$ $\bar{y} = \frac{1\frac{4}{9}}{2} = \frac{13}{18} \quad (= 0.722 \dots)$	<p>B1</p> <p>M1A1</p> <p>M1A1</p> <p>M1A1</p> <p>DM1A1 (9)</p>
	<p>B1 for a correct area of <math>R</math> (may be embedded in the working)</p> <p>M1 attempting the integral <math>\int_1^3 xy \, dx</math> (integration to be seen)</p> <p>A1 correct integration and limits (substitution not needed)</p> <p>M1 divide by their area - denominator <b>must</b> be an area</p> <p>A1 correct value for <math>\bar{x}</math> - can be exact or decimal 1.6 or better</p> <p>M1 attempting the integral <math>\int_1^3 \frac{1}{2} y^2 \, dx</math> or <math>\int_1^3 y^2 \, dx</math> (integration to be seen)</p> <p>A1 correct integration (of their integral) and limits shown</p> <p>DM1 divide by their area <b>must</b> have used <math>\int \frac{1}{2} y^2 \, dx</math></p> <p>A1 correct value for <math>\bar{y}</math> - can be exact or decimal 0.72 or better</p>	

Question Number	Scheme	Marks
3	$2T \cos 30 = T \cos 30 + mg$ $\frac{T\sqrt{3}}{2} = mg$ $3T \cos 60 = mr\omega^2$ $\frac{3}{2} \times \frac{2mg}{\sqrt{3}} = mr\omega^2$ $AB = 4a \quad \frac{r}{2a} = \tan 30 = \frac{1}{\sqrt{3}}$ $r = \frac{2a}{\sqrt{3}}$ $\frac{3g}{\sqrt{3}} = \frac{2a}{\sqrt{3}} \omega^2$ $\omega^2 = \frac{3g}{2a} \quad \omega = \sqrt{\frac{3g}{2a}}$	<p>M1A1</p> <p>A1</p> <p>M1A1</p> <p>A1</p> <p>DM1 Dep on both prev M marks</p> <p>A1</p> <p>[8]</p>
	<p>M1 resolve vertically - both tensions resolved</p> <p>A1 fully correct equation</p> <p>A1 substitute for trig function</p> <p>M1 NL2 horizontally - tensions resolved, acceleration in either form</p> <p>A1 correct equation, <math>r</math> still present, acceleration <math>r\omega^2</math></p> <p>A1 correct equation with no trig function</p> <p>DM1 eliminate <math>r</math> and <math>T</math> to obtain an equation with <math>\omega</math>, <math>a</math>, <math>g</math> and no other letters <math>r</math> need not be correct but do not allow <math>r = a</math></p> <p>A1 correct result</p> <p>For first 6 marks the equations can have <math>T</math> and <math>2T</math> or two different tensions</p>	

Question Number	Scheme	Marks
<p><b>4</b></p> <p><b>(a)</b></p>	<p>Length of string/half string = 10 m / 5 m (or extn = 5 m)</p> $T = \frac{\lambda x}{l} = \frac{20 \times 5}{5}, = 20$ $2T \cos \alpha = mg$ $2 \times 20 \times \frac{4}{5} = mg$ <p>Weight = 32 N (Accept <math>mg = 32</math>)</p> <p><b>(b)</b></p> <p>PE lost = "<math>mg</math>" <math>\times 4</math></p> $\text{EPE gained} = \frac{20 \times 5^2}{2 \times 5} - \frac{20 \times 1^2}{2 \times 5}$ $\frac{1}{2}mv^2 = "mg" \times 4 - \left( \frac{20 \times 5^2}{2 \times 5} - \frac{20 \times 1^2}{2 \times 5} \right)$ $\frac{16}{g}v^2 = 32 \times 4 - \left( \frac{20 \times 5^2}{2 \times 5} - \frac{20 \times 1^2}{2 \times 5} \right)$ $v^2 = 5g$ <p><math>v = 7, 7.0 \text{ or } 7.00</math></p>	<p>B1</p> <p>M1, A1</p> <p>M1</p> <p>A1ft</p> <p>A1 (6)</p> <p>M1A1A1</p> <p>DM1</p> <p>A1 (5) [11]</p>
<p><b>(a)</b></p> <p><b>(b)</b></p>	<p>B1 correct length of complete or half string or correct extension (need not be shown explicitly)</p> <p>M1 apply Hooke's law <math>x \neq 1</math></p> <p>A1 correct tension obtained</p> <p>M1 resolving vertically, both tensions resolved</p> <p>A1ft substitute their tension and <math>\cos \alpha = \frac{4}{5}</math></p> <p>A1 correct weight obtained (no ft)</p> <p>M1 energy equation with KE, PE and two EPE terms - all calculated with correct formulae</p> <p>A1A1 Deduct one A mark per error (if <math>m</math> is substituted early, ft their <math>m</math>)</p> <p>M1 Substitute their mass (not weight)</p> <p>A1 correct value for <math>v</math> 7, 7.0 or 7.00 only acceptable</p>	

Question Number	Scheme	Marks
<b>5 (a)</b>	<div> <div>Small cone</div> <div>Large cone</div> <div><math>S</math></div> </div> <div> <div>Mass</div> <div><math>\frac{4}{3}\pi r^3\rho</math></div> <div><math>\frac{1}{3}k\pi r^3\rho</math></div> <div><math>\frac{1}{3}\pi r^3\rho(4+k)</math></div> </div> <div> <div>Ratio</div> <div>4</div> <div><math>k</math></div> <div><math>4+k</math></div> </div> <div> <div>Disp from <math>O</math></div> <div><math>-r</math></div> <div><math>\frac{kr}{4}</math></div> <div><math>\bar{x}</math></div> </div> <div> <math display="block">-4r + \frac{k^2r}{4} = (4+k)\bar{x}</math> <math display="block">\bar{x} = \frac{(k^2-16)r}{4(4+k)} = \frac{1}{4}(k-4)r</math> </div>	<div>M1A1A1</div> <div>A1 (4)</div>
<b>(b)</b>	<div><math>k</math> greatest when <math>\frac{\bar{x}}{r} = \frac{r}{4r}</math></div> <div><math>\frac{1}{4}(k-4) = \frac{1}{4}</math></div> <div>Greatest <math>k = 5</math></div>	<div>M1</div> <div>A1ft</div> <div>A1 (3)</div>
<b>(c)</b>	<div><math>\tan 12^\circ = \frac{\bar{x}}{r} = \frac{1}{4}(k-4)</math></div> <div><math>k = 4.85 \quad 4.9 \quad (4.8502\dots)</math></div>	<div>M1A1ft</div> <div>A1 (3)</div> <div>[10]</div>
<b>(a)</b>	<div>M1 moments equation about any (suitable) point. Volumes or ratio of volumes used.</div> <div>A1 LHS correct</div> <div>A1 RHS side</div> <div>A1 correct distance from <math>O</math>, inc use of <math>k &gt; 4</math> Single fraction only in the expression</div>	
<b>(b)</b>	<div>M1 using vertical through c of m passes through A to obtain a connection between <math>\bar{x}</math> and <math>r</math> or a numerical value for <math>\bar{x}</math> or any other complete valid method</div> <div>A1ft obtain a correct equation for <math>k</math> with their <math>\bar{x}</math></div> <div>A1 cao <math>k = 5</math> (inequality gets A0)</div>	
<b>(c)</b>	<div>M1 <math>\tan 12^\circ = \frac{\bar{x}}{r}</math> either way up</div> <div>A1ft substitute for <math>\bar{x}</math> correct way up now</div> <div>A1 Final answer 4.9, 4.85 or better</div>	

Question Number	Scheme	Marks
<b>6</b>		
<b>(a)</b>	$\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{ag}{5}\right) = mga(1 - \cos \theta)$ $v^2 = 2ag + \frac{ag}{5} - 2ag \cos \theta = \frac{ag}{5}(11 - 10 \cos \theta) \quad *$	M1A1A1  A1 (4)
<b>(b)</b>	$mg \cos \alpha (-R) = m \frac{v^2}{a}$ $g \cos \alpha = \frac{g}{5}(11 - 10 \cos \alpha) \quad \text{or sub } \cos \alpha = \frac{v^2}{ag} \text{ in energy equation}$ $\cos \alpha = \frac{11}{15}$ $P \text{ leaves the sphere with speed } \sqrt{\frac{ag}{5}\left(11 - \frac{22}{3}\right)} = \sqrt{\frac{11ag}{15}}$	M1A1  M1 A1  DM1A1 (6)
<b>(c)</b>	$\text{Horiz comp} = \sqrt{\frac{11ag}{15}} \times \cos \alpha = \sqrt{\frac{11ag}{15}} \times \frac{11}{15}$ $\text{By cons of energy from top: } 2mag = \frac{1}{2}mV^2 - \frac{1}{2}m\frac{ag}{5}$ $V^2 = \frac{21ag}{5}$ $\cos \theta = \sqrt{\frac{11ag}{15}} \times \frac{11}{15} \times \sqrt{\frac{5}{21ag}} = \sqrt{\frac{11}{63}} \times \frac{11}{15} = 0.30642...$ $\theta = 72.155... \quad \text{Accept } 72^\circ \text{ or better}$	M1  M1  A1  M1  A1 (5)
		[15]
<b>(a)</b>	<p>M1 <b>Energy</b> equation from start to general position - must have 2 KE terms and a loss of PE</p> <p>A1 LHS correct</p> <p>A1 RHS correct</p> <p>A1cso re-arrange to the given result</p>	



Question Number	Scheme	Marks
(b)	<p>M1 NL2 along radius, acceleration in either form, <math>R</math> need not be shown, weight must be resolved</p> <p>A1 fully correct equation with or w/o <math>R</math>, accel now <math>\frac{v^2}{a}</math></p> <p>M1 elimination of <math>v^2</math> or <math>\cos \alpha</math></p> <p>A1 correct equation after elimination</p> <p>DM1 substitute their <math>\cos \alpha</math> to obtain an expression for <math>v^2</math> Dep on both previous M marks</p> <p>A1 correct expression for <math>v</math></p>	
	<p>(c)</p> <p>M1 obtaining an expression for the horiz comp of speed at <math>P</math></p> <p>M1 use energy to obtain the speed when particle hits the floor</p> <p>A1 correct speed at floor</p> <p>M1 use horizontal speed and resultant speed to find the angle</p> <p>A1 correct angle 2 sf or more figures (<math>g</math> cancels)</p> <p>ALT: By SUVAT:</p> <p>M1 Horiz component</p> <p>M1 Vert component and complete attempt for speed at the floor (including the vertical height)</p> <p>A1 correct vert speed at floor</p> <p>M1 attempt angle using <math>\tan</math> (either way up)</p> <p>A1 correct angle 2 sf or more figures (<math>g</math> cancels)</p>	

Question Number	Scheme	Marks
<b>7</b>		
<b>(a)</b>	$T = \frac{\lambda a / 5}{a}$ $T = mg \cos 60 = \frac{1}{2} mg$ $\frac{1}{2} mg = \frac{\lambda}{5} \quad \lambda = \frac{5}{2} mg \quad *$	M1A1
<b>(b)</b>	<p>When string has length <math>\left(\frac{6a}{5} + x\right)</math>:</p> $\frac{1}{2} mg - \frac{5}{2} mg \left(\frac{a/5 + x}{a}\right) = m\ddot{x}$ $-\frac{5g}{2a} x = \ddot{x}, \Rightarrow \text{SHM}$ $\text{Period} = 2\pi \sqrt{\frac{2a}{5g}} \quad *$	M1A1A1 DM1,A1 A1 (6)
<b>(c)</b>	$\text{Max accel} = \omega^2 \times \text{amp} = \omega^2 \frac{a}{5} = \frac{5g}{2a} \times \frac{a}{5} = \frac{g}{2}$	M1A1 (2)
<b>(d)</b>	$x = \frac{a}{5} \sin \omega t$ $\frac{a}{10} = \frac{a}{5} \sin \omega t$ $\omega t = \sin^{-1} 0.5 = \frac{\pi}{6}$ $t = \frac{\pi}{6\omega} = \frac{\pi}{6} \sqrt{\frac{2a}{5g}}$ $\text{Total time} = t = \frac{\pi}{6} \sqrt{\frac{2a}{5g}} + \frac{\pi}{2} \sqrt{\frac{2a}{5g}} = \frac{2\pi}{3} \sqrt{\frac{2a}{5g}}$	M1 A1 M1A1 (4) [16]

Question Number	Scheme	Marks
(a)	<p>M1 Hooke's Law used to find <math>T</math> at <math>B</math></p> <p>A1 correct equation</p> <p>M1 eliminating <math>T</math> by use of resolving along the plane</p> <p>A1 also correct value for <math>\lambda</math></p>	
(b)	<p>M1 NL2 along the plane when extension is <math>\frac{a}{5} + x</math> - must have a tension and a component of the weight. Allow with <math>\ddot{x}</math> or <math>f</math> (acceleration).</p> <p>A1A1 deduct one per error. (difference of forces wrong way round is one error only) mass <math>\times</math> acceleration (not <math>\ddot{x}</math>) is also an error</p> <p>DM1 simplify to the correct form acceleration must be <math>\ddot{x}</math> now</p> <p>A1 also correct final equation AND conclusion</p> <p>A1 correct period</p>	
(c)	<p>M1 obtaining the max acceleration, <math>\text{amp} \neq a</math></p> <p>A1 correct max acceleration (no ft)</p>	
(d)	<p>M1 using equation for <math>x</math> - sin or cos form and solving for <math>t</math> - must use radians and <math>\omega = \sqrt{\frac{5g}{2a}}</math> <math>\text{amp} \neq a</math></p> <p>A1 correct value for <math>t</math> from their equation</p> <p>M1 complete to obtain the required time</p> <p>A1 correct total time</p> <p>If time from end point to <math>x = -\frac{a}{10}</math> is found mark M1M1A1A1</p>	





Mark Scheme (Results)

Summer 2015

Pearson Edexcel International A Level  
in Mechanics 3 (WME03/01)

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Summer 2015

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**PEARSON EDEXCEL IAL MATHEMATICS****General Instructions for Marking**

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

**'M' marks**

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.

e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

**'A' marks**

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

**'B' marks**

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.



### 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\surd$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - ☐ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Mechanics Marking

*(But note that specific mark schemes may sometimes override these general principles)*

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra  $g$  in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of  $g = 9.8$  should be given to 2 or 3 SF.
- Use of  $g = 9.81$  should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

SHM Simple harmonic motion

PCLM Principle of conservation of linear momentum

RHS, LHS Right hand side, left hand side.

June 2015  
WME03 M3  
Mark Scheme

Question Number	Scheme	Marks
<b>1</b>	<p>(30° or <math>\theta</math> for the first 3 lines)</p> $R \sin 30^\circ = mg$ $R \cos 30^\circ = m(r \cos 30^\circ) \omega^2$ $\omega^2 = \frac{R}{mr} = \frac{g}{r \sin 30}$ $\omega = \sqrt{\frac{2g}{r}}$ $\text{Time} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r}{2g}} = \pi \sqrt{\frac{2r}{g}} \quad *$	<p>M1A1</p> <p>M1A1A1</p> <p>DM1</p> <p>A1</p> <p>A1cso [8]</p>

**M1** Resolving vertically 30° or  $\theta$

**A1** Correct equation 30° or  $\theta$

**M1** Attempting an equation of motion along the radius, acceleration in either form 30° or  $\theta$   
Allow with  $r$  for radius

**A1** LHS correct 30° or  $\theta$

**A1** RHS correct, 30° or  $\theta$  but not  $r$  for radius

**DM1** Obtaining an expression for  $\omega^2$  or for  $v^2$  **and** the length of the path 30° or  $\theta$  Dependent on both previous M marks

**A1** Correct expression for  $\omega$  Must have the numerical value for the trig function now

**A1cso** Deducing the GIVEN answer

**ALT:** Resolve perpendicular to the reaction:

$$mg \cos 30 = m \times rad \times \omega^2 \cos 60$$

$$= mr \cos 30 \omega^2 \cos 60$$

Obtain  $\omega$

Correct time

M2A1(LHS) A1(RHS)

A1

M1A1

A1

Question Number	Scheme	Marks
<b>2 (a)</b>	$F = \frac{K}{x^2}$ $x = R \Rightarrow F = mg \quad \therefore mg = \frac{K}{R^2}$ $K = mgR^2 \quad *$	<p>M1</p> <p>A1 (2)</p>
<b>(b)</b>	$\frac{mgR^2}{x^2} = -mv \frac{dv}{dx}$ $g \int \frac{R^2}{x^2} dx = - \int v dv$ $-g \frac{R^2}{x} = -\frac{1}{2} v^2 \quad (+c)$ $x = 3R, v = V \Rightarrow -g \frac{R^2}{3R} = -\frac{1}{2} V^2 + c$ $c = -\frac{Rg}{3} + \frac{1}{2} V^2$ $x = R \Rightarrow \frac{1}{2} v^2 = -\frac{Rg}{3} + \frac{1}{2} V^2 + g \frac{R^2}{R}$ $v^2 = V^2 + \frac{4Rg}{3}$ $v = \sqrt{V^2 + \frac{4Rg}{3}}$	<p>M1</p> <p>DM1A1ft</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 cso (7)</p> <p>[9]</p>

- (a) M1** Setting  $F = mg$  **and**  $x = R$   
**A1** Deducing the GIVEN answer

- (b) M1** Attempting an equation of motion with acceleration in the form  $v \frac{dv}{dx}$ . The minus sign may be missing.  
**DM1** Attempting the integration  
**A1ft** Correct integration, follow through on a missing minus sign from line 1, constant of integration may be missing  
**M1** Substituting  $x = 3R, v = V$  to obtain an equation for  $c$   
**A1** Correct expression for  $c$   
**M1** Substituting  $x = R$  and their expression for  $c$   
**A1** Correct expression for  $v$ , any equivalent form

Question Number	Scheme	Marks
<b>3 (a)</b>	$\frac{dv}{dt} = -2(t+4)^{-\frac{1}{2}}$ $v = -\int 2(t+4)^{-\frac{1}{2}} dt$ $v = -4(t+4)^{\frac{1}{2}} \quad (+c)$ $t = 0, v = 8 \Rightarrow c = 16$ $v = 16 - 4(t+4)^{\frac{1}{2}} \quad (\text{m s}^{-1}) \quad *$	<p>M1</p> <p>DM1A1</p> <p>M1</p> <p>A1cso (5)</p>
<b>(b)</b>	$v = 0 \quad 16 = 4(t+4)^{\frac{1}{2}}$ $16 = t + 4 \quad t = 12$ $x = 4 \int \left( 4 - (t+4)^{\frac{1}{2}} \right) dt$ $x = 4 \left( 4t - \frac{2}{3}(t+4)^{\frac{3}{2}} \right) \quad (+d)$ $t = 0, x = 0 \quad d = 4 \times \frac{2}{3} \times 4^{\frac{3}{2}} = \frac{64}{3} \quad \text{oe}$ $t = 12 \quad x = 4 \left( 4 \times 12 - \frac{2}{3} \times 16^{\frac{3}{2}} \right) + \frac{64}{3} = 42 \frac{2}{3} \quad (\text{m}) \quad \text{oe eg 43 or better}$	<p>M1</p> <p>A1</p> <p>M1A1</p> <p>A1</p> <p>DM1A1 (7)</p>

[12]

(a) **M1** Attempting an expression for the acceleration in the form  $\frac{dv}{dt}$ ; minus may be omitted.

**DM1** Attempting the integration

**A1** Correct integration, constant of integration may be omitted (no ft)

**M1** Using the initial conditions to obtain a value for the constant of integration

**Also** Substitute the value of  $c$  and obtain the final **GIVEN** answer

**(b) M1** Setting the **given** expression for  $v$  equal to 0

**A1** Solving to get  $t = 12$

**M1** Setting  $v = \frac{dx}{dt}$  and attempting the integration wrt  $t$  At least one term must clearly be

Integrated.

**A1** Correct integration, constant may be omitted

**A1** Substituting  $t = 0$ ,  $x = 0$  and obtaining the correct value of  $d$ . Any equivalent number, in decimals.

**DM1** Substituting their value for  $t$  and obtaining a value for the required distance. Dependent on The second M mark.

**A1** Correct final answer, any equivalent form.

Question Number	Scheme	Marks
<b>4(a)</b>	Energy to top: $\frac{1}{2} \times 3m \times u^2 - \frac{1}{2} \times 3mv^2 = 3mga$ NL2 at top: $T + 3mg = 3m \frac{v^2}{a}$ $T = 3m \frac{u^2}{a} - 6mg - 3mg$ $T \geq 0 \Rightarrow \frac{u^2}{a} \geq 3g$ $u^2 \geq 3ag$ *	M1A1 M1A1 DM1 M1 A1 cso (7)
<b>(b)</b>	Tension at bottom: $\frac{1}{2} \times 3m \times V^2 - \frac{1}{2} \times 3mu^2 = 3mga$ $T_{\max} - 3mg = 3m \frac{V^2}{a}$ $T_{\max} = 3mg + 6mg + 3m \frac{u^2}{a}$ $T_{\min} = 3m \frac{u^2}{a} - 9mg$ $9mg + 3m \frac{u^2}{a} = 3 \left( 3m \frac{u^2}{a} - 9mg \right)$ $u^2 = 6ag$ *	M1 M1 A1 DM1 A1 cso (5) [12]

- (a) M1** Attempting an energy equation, can be to a general point for this mark. Mass can be missing but use of  $v^2 = u^2 + 2as$  scores M0
- A1** Correct equation from A to the top
- M1** Attempting an equation of motion along the radius at the top, acceleration in either form
- A1** Correct equation, acceleration in form  $\frac{v^2}{r}$
- DM1** Eliminate  $v^2$  to obtain an expression for  $T$  Dependent on both previous M marks
- M1** Use  $T \geq 0$  at top to obtain an inequality connecting  $a$ ,  $g$  and  $u$
- A1cso** Re-arrange to obtain the GIVEN answer
- (b) M1** Attempting an energy equation to the bottom, maybe from A or from the top
- M1** Attempting an equation of motion along the radius at the bottom
- A1** Correct expression for the max tension
- DM1** Forming an equation connecting *their* tension at the top with *their* tension at the bottom. If the 3 is multiplying the wrong tension this mark can still be gained. Dependent on both previous M marks
- A1cso** Obtaining the GIVEN answer.

Question Number	Scheme	Marks
<b>5 (a)</b>	$T = \frac{20e}{2} = \frac{15(1.8 - e)}{1.2}$ $10e \times 1.2 = 15(1.8 - e)$ $e = 1$ $AO = 3 \text{ m} \quad *$	M1A1 A1 A1cso (4)
<b>(b)</b>	$0.5\ddot{x} = \frac{20(1-x)}{2} - \frac{15(0.8+x)}{1.2}$ $\ddot{x} = -45x \quad \therefore \text{SHM}$	M1A1A1 A1cso (4)
<b>(c)</b>	<p>String becomes slack when <math>x = (-)0.8</math> (allow wo sign due to symmetry)</p> $v^2 = \omega^2(a^2 - x^2)$ $v^2 = 45(1 - 0.8^2) \quad (=16.2)$ $v = 4.024... \text{ m s}^{-1} \quad (4.0 \text{ or better})$	B1 M1A1ft A1ft (4)
<b>(d)</b>	$\frac{1}{2} \times \frac{20y^2}{2} - \frac{1}{2} \times \frac{20 \times 1.8^2}{2} = \frac{1}{2} \times 0.5 \times 16.2 \quad \text{ft on } v$ $20y^2 - 64.8 = 16.2$ $y^2 = 4.05 \quad y = 2.012...$ $\text{Distance } DB =  5 - 4.012...  = 0.988... \text{ m} \quad (\text{accept } 0.99 \text{ or better})$	M1A1A1ft A1 A1ft (5) [17]
Alt for d:	<p>Prove SHM with only one string      M1A1 (equation)</p> <p>Value <math>\omega</math>      A1</p> <p>Use <math>v = a\omega</math> to find <math>a</math>      A1ft (ft on <math>v</math>)</p> <p>Dist      A1ft</p>	

- (a) M1** Attempting to obtain and equate the tensions in the two parts of the string.  
**A1** Correct equation, extension in  $AP$  or  $BP$  can be used or use  $OA$  as the unknown  
**A1** Obtaining the correct extension in either string (ext in  $BP = 0.8 \text{ m}$ ) or another useful distance  
**A1cso** Obtaining the correct GIVEN answer
- (b) M1** Forming an equation of motion at a general point. There must be a difference of tensions, both with the variable. May have  $m$  instead of  $0.5$  Accel can be  $a$   
**A1 A1** Deduct 1 for each error,  $m$  or  $0.5$  allowed, acceleration to be  $\ddot{x}$  now  
**A1cso** Correct equation in the required form, with a concluding statement;  $m$  or  $0.5$  allowed
- (c) B1** For  $x = \pm 0.8$  Need not be shown explicitly  
**M1** Using  $v^2 = \omega^2(a^2 - x^2)$  with *their* (numerical)  $\omega$  and their  $x$   
**A1ft** Equation with correct numbers ft their  $\omega$   
**A1ft** Correct value for  $v$  2sf or better or exact
- (d) M1** Attempting an energy equation with 2 EPE terms and a KE term  
**A1** 2 correct terms may have  $(1.8 + x)$  instead of  $y$   
**A1ft** Completely correct equation, follow through their  $v$  from (c)  
**A1** Correct value for distance travelled after  $PB$  became slack.  $x = 0.21$   
**A1ft** Complete to the distance  $DB$ . Follow through their distance travelled after  $PB$  became slack.  
**Alternatives at end of mark scheme**

Question Number	Scheme	Marks
<b>6(a)</b>	$\text{Vol} = \pi \int_0^2 (x^2 + 3)^2 dx$ $= \pi \int_0^2 (x^4 + 6x^2 + 9) dx$ $= \pi \left[ \frac{1}{5}x^5 + 2x^3 + 9x \right]_0^2$ $= \frac{202}{5} \pi \quad \text{cm}^3 \quad *$	<p>M1</p> <p>DM1A1</p> <p>A1 (4)</p>
<b>(b)</b>	$\pi \int_0^2 x(x^2 + 3)^2 dx = \pi \int_0^2 (x^5 + 6x^3 + 9x) dx$ $= \pi \left[ \frac{1}{6}x^6 + \frac{3}{2}x^4 + \frac{9}{2}x^2 \right]_0^2$ $= \frac{158}{3} \pi$ <p>(Or by chain rule or substitution)</p> $\text{C of m} = \frac{158}{3} \times \frac{5}{202}, = 1.3036... = 1.30 \quad \text{cm}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1A1 (5)</p>
<b>(c)</b>	<p>Mass ratio <math>2 \times \frac{202}{5} \pi \quad \frac{1}{3} \pi \times 7^2 \times 6 \quad \left( \frac{404}{5} + 98 \right) \pi</math></p> <p>Dist from V <math>6.7 \quad 4.5 \quad \bar{x}</math></p> $\frac{404}{5} \times 6.7 + 98 \times 4.5 = \left( \frac{404}{5} + 98 \right) \bar{x}$ $\bar{x} = \frac{\frac{404}{5} \times 6.7 + 98 \times 4.5}{\left( \frac{404}{5} + 98 \right)} = 5.494... = 5.5 \text{ cm} \quad \text{Accept 5.49 or better}$	<p>B1</p> <p>B1</p> <p>M1A1ft</p> <p>A1 (5)</p>
<b>(d)</b>	$\tan \theta = \frac{6 - \bar{x}}{7} = \frac{0.5058...}{7}$ $\alpha = \tan^{-1} \left( \frac{6}{7} \right) - \tan^{-1} \left( \frac{0.5058...}{7} \right) = 36.468...^\circ = 36^\circ \quad \text{or better}$	<p>M1</p> <p>M1A1 (3) [17]</p>



Question Number	Scheme	Marks
(a) <b>M1</b>	Using $\pi \int y^2 dx$ with the equation of the curve, no limits needed	
<b>DM1</b>	Integrating their expression for the volume	
<b>A1</b>	Correct integration inc limits now	
<b>A1</b>	Substituting the limits to obtain the GIVEN answer	
(b) <b>M1</b>	Using $(\pi) \int xy^2 dx$ with the equation of the curve, no limits needed, $\pi$ can be omitted	
<b>A1</b>	Correct integration, including limits; no substitution needed for this mark	
<b>A1</b>	Correct substitution of limits	
<b>M1</b>	Use of $\frac{\pi \int xy^2 dx}{\pi \int y^2 dx}$ with their $\pi \int xy^2 dx$ . $\pi$ must be seen in both numerator and denominator or in neither.	
<b>A1cao</b>	Correct answer. Must be 1.30	
(c) <b>B1</b>	Correct mass ratio	
<b>B1</b>	Correct distances, from V or any other point, provided consistent	
<b>M1</b>	Attempting a moments equation	
<b>A1ft</b>	Correct equation, follow through their distances and mass ratio	
<b>A1</b>	Correct distance from V	
(d) <b>M1</b>	Attempting the tan of an appropriate angle, numbers either way up	
<b>M1</b>	Attempting to obtain the required angle	
<b>A1</b>	Correct final answer 2sf or more	

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**Alternatives for 5(d)**


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**1** EPE in BP (at release) transferred to EPE in AP (same as MS, except 1 term for first A1)

**2**  $0.5a = -10(1.8 + x)$

$$v \frac{dv}{dx} = -36 - 10x$$

$$\int v dv = - \int (36 + 10x) dx$$

$$\frac{v^2}{2} = -36x + 5x^2 + c$$

M1A1

$$x = 0, v = \frac{9\sqrt{5}}{5} \therefore c = 8.1$$

A1

Then  $v = 0$  etc

M1A1



# Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in Mechanics 3 (6679/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

#### 'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.

e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

#### 'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

#### 'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

### 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
  - 5 For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  - 6 If a candidate makes more than one attempt at any question:
    - a. If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - b. If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
  - 7 Ignore wrong working or incorrect statements following a correct answer.



## General Principles for Mechanics Marking

*(But note that specific mark schemes may sometimes override these general principles)*

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra  $g$  in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- dM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of  $g = 9.8$  should be given to 2 or 3 SF.
- Use of  $g = 9.81$  should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

SHM Simple harmonic motion

PCLM Principle of conservation of linear momentum

RHS, LHS Right hand side, left hand side.

June 2015  
6679 M3  
Mark Scheme

Question Number	Scheme	Marks
<b>1</b>		
<b>(a)</b>	$0.5g = T = \frac{\lambda \times 0.3}{1.2}$ $\lambda = 2g = 19.6$	M1A1  A1 (3)
<b>(b)</b>	$\frac{1}{2} \times \frac{19.6 \times x^2}{1.2} - \frac{1}{2} \times \frac{19.6 \times 0.4^2}{1.2} = 0.5 \times g \times (x + 0.4)$ $5x^2 - 3x - 2 = 0$ $(5x + 2)(x - 1) = 0 \quad \text{or use of diff of 2 squares to obtain and then solve a linear equation}$ $x = 1 \quad (x = -0.4 \text{ need not be seen})$ $AC = 2.2 \text{ m}$	M1A1ftA1       A1 (4) [7]

**(a) M1** Use Hooke's law to obtain the tension and equate to the weight

**A1** Correct equation

**A1** Solve to get  $\lambda = 19.6$  Accept 20 or  $2g$

**(b) M1** Attempt an energy equation with the difference of 2 EPE terms and a loss of GPE

EPE formula must be of the form  $k \frac{\lambda x^2}{l}$

**A1ft** EPE terms correct follow through their  $\lambda$

**A1** GPE term correct, including all signs in the equation correct If  $x$  used for EPE and GPE A0 here

**A1** Correct length  $AC$  If  $\lambda = 20$  is used, this is p.a. and so scores A0

**ALT:** Find  $BC$  first:  $\frac{1}{2} \times \frac{19.6 \times (h - 0.4)^2}{1.2} - \frac{1}{2} \times \frac{19.6 \times 0.4^2}{1.2} = 0.5gh$  M1A1A1  
 $BC = 1.4$   $AC = 2.2$  A1

Methods depending on SHM must prove SHM first, but if correct answer only is given award B1 (M1 on e-PEN)

By integration: Integrating and substituting yields an equation equivalent to the one shown - mark from here M1A1A1ft -1 each error ft on  $\lambda$

Question Number	Scheme	Marks
<b>2 (a)</b>	$\text{Vol} = \pi \int_0^1 4e^{2x} dx$ $= \pi \left[ 2e^{2x} \right]_0^1$ $= 2\pi(e^2 - 1) \quad *$	<b>M1</b> <b>DM1A1</b> <b>A1cso</b> (4)
<b>(b)</b>	$\pi \int_0^1 4xe^{2x} dx$ $= 4\pi \left\{ \left[ x \times \frac{1}{2} e^{2x} \right]_0^1 - \int_0^1 \frac{1}{2} e^{2x} dx \right\}$ $= 4\pi \left[ \frac{1}{2} e^2 - 0 \right] - 4\pi \left[ \frac{1}{4} e^{2x} \right]_0^1$ $= \pi(e^2 + 1)$ $x \text{ coord} = \frac{\pi(e^2 + 1)}{2\pi(e^2 - 1)}, = \frac{e^2 + 1}{2(e^2 - 1)} \quad \text{oe}$	<b>M1</b> <b>DM1</b> <b>A1</b> <b>A1</b> <b>M1A1</b> (6) [10]

**(a) M1** Using  $\pi \int y^2 dx$  with the equation of the curve, no limits needed

**DM1** Integrating their expression for the volume

**A1** Correct integration inc limits now

**A1** Substituting the limits to obtain the GIVEN answer

**(b) M1** Using  $(\pi) \int xy^2 dx$  with the equation of the curve, no limits needed,  $\pi$  can be omitted

**DM1** Attempting to use integration by parts; allow  $\pm$  between the two parts. No limits needed

**A1** Correct integration, including limits; no substitution needed for this mark

**A1** Correct after limits substituted

**M1** Use of  $\frac{\pi \int xy^2 dx}{\pi \int y^2 dx}$  with their  $\pi \int xy^2 dx$ .  $\pi$  must be seen in both numerator and

denominator or in neither. This mark is not dependent on the previous M marks

**A1cao** Correct answer.

Question Number	Scheme	Marks
<b>3(a)</b>	$R(\uparrow) \quad T_A \cos 30 = mg + T_B \cos 30$ $NL2 \quad T_A \cos 60 + T_B \cos 60 = mr\omega^2$ $= m \times 2l \cos 60 \omega^2 \quad \text{or} \quad ml\omega^2$ $T_A + T_B = 2ml\omega^2$ $T_A - T_B = \frac{2mg}{\sqrt{3}}$	M1A1  M1A1  A1
<b>(i)</b>	$T_A = \frac{m}{3}(3l\omega^2 + g\sqrt{3}) \quad \text{oe}$	DM1A1
<b>(ii)</b>	$T_B = \frac{m}{3}(3l\omega^2 - g\sqrt{3}) \quad \text{oe}$	A1 (8)
<b>(b)</b>	$T_B \geq 0 \Rightarrow 3l\omega^2 \geq g\sqrt{3}$	M1
	$\omega^2 \geq \frac{g\sqrt{3}}{3l} \quad *$	A1cso (2)
		[10]

**(a) M1** Resolving vertically

**A1** Correct equation

**M1** NL2 along radius, acceleration in either form

**A1** LHS correct

**A1** Correct radius substituted and accel in  $r\omega^2$ . Can be awarded later by implication if work implies correct radius used.

**DM1** Solving the two equations to obtain an expression for either tension. Dependent on both previous M marks

**A1** Tension in AP correct – simplified to two terms

**A1** Tension in BP correct – simplified to two terms

**(b) M1** Using their tension in  $BP \geq 0$  **must be**  $\geq$  for this mark

**A1cso** Obtaining the GIVEN answer. Only error allowed is the expression for the tension in AP

Question Number	Scheme	Marks												
4(a)	$\frac{63000}{kt^2} = 900 \frac{dv}{dt}$	M1												
	$-\frac{70}{kt} (+c) = v$	DM1A1ft												
	$t = 1 \quad v = 0 \Rightarrow c = \frac{70}{k}$	M1(either)												
	$t = 4 \quad v = 10.5 \Rightarrow -\frac{70}{4k} + c = 10.5$	A1(both)												
	$-\frac{70}{4k} + \frac{70}{k} = 10.5$													
	$k = 5, \quad c = 14$	A1												
	$v = 14 - \frac{14}{t} \quad *$	A1 cso (7)												
(b)	$\frac{14}{t} > 0 \Rightarrow v < 14$ or $v$ never reaches 14	B1 (1)												
(c)	$7 = 14 - \frac{14}{t}$													
	$\frac{14}{t} = 7 \quad t = 2$	B1												
	<table><tr><td><math>t</math></td><td>1</td><td>1.25</td><td>1.5</td><td>1.75</td><td>(2)</td></tr><tr><td><math>v</math></td><td>0</td><td>2.8</td><td>4.666..</td><td>6</td><td>7</td></tr></table>	$t$	1	1.25	1.5	1.75	(2)	$v$	0	2.8	4.666..	6	7	
	$t$	1	1.25	1.5	1.75	(2)								
	$v$	0	2.8	4.666..	6	7								
$x = \frac{0.25}{2} (0 + 2 \times 2.8 + 2 \times 4.666... + 2 \times 6 + 7)$	M1A1													
$X = 4.24175$ Accept 4.2 or 4.24	A1 (4) [12]													

**(a) M1** Forming an equation of motion with acceleration as  $\frac{dv}{dt} = 900$  or  $m$

**DM1** Attempting the integration

**A1** Correct equation. Constant of integration not needed

**M1** Substituting either pair of given values

**A1** Obtaining correct equations using each pair of values

**A1** Obtaining correct values for  $c$  **and**  $k$  or use  $k=5, \quad c = \frac{70}{k}$

**A1** Substituting these values to obtain the GIVEN answer  
Misread eg 6300 for 63000: M1DM1A1M1A0A0A0

**(b) B1** Must be clear that  $v < 14$  not just never = 14  $\frac{14}{t} > 0$  essential

**(c) B1** Showing that  $t = 2$  when  $v = 7$  Award if seen as upper limit for  $t$  in trapezium rule or values 1.25, 1.5, 1.75 seen for  $t$

**M1** Using the trapezium rule. Must have 4 intervals and values of  $t$  shown in the table.

**A1** Correct numbers in the trapezium rule statement.

Values of  $v$  can be in the form  $14 - \frac{14}{1.25}$  etc

**A1** Correct final answer. It is an estimate, so 2 or 3 sf only.

Question Number	Scheme	Marks
<b>5</b>	<p>Dist of c of m from <math>O = r \tan 30 = \frac{r}{\sqrt{3}}</math></p> <p>Ratio of masses <math>\frac{M}{1} \quad \frac{kM}{k} \quad \frac{(1+k)M}{1+k}</math></p> <p>Dist from <math>O \quad -\frac{1}{4}h \quad \frac{kh}{4} \quad \frac{r}{\sqrt{3}}</math></p> <p><math>M(O) \quad -\frac{1}{4}h + \frac{k^2h}{4} = (1+k)\frac{r}{\sqrt{3}}</math></p> <p><math>\frac{h}{4}(k^2 - 1) = (k+1)\frac{r}{\sqrt{3}}</math></p> <p><math>k = \frac{4r}{h\sqrt{3}} + 1 \quad *</math></p>	<p>M1A1</p> <p>M1A1A1ft</p> <p>A1 [6]</p>
<b>Alt 1</b>	<p>By moments about A</p> <p><math>kMg\left(\frac{1}{4}kh \cos 30 - r \sin 30\right), \quad Mg\left(\frac{1}{4}h \cos 30 + r \sin 30\right)</math></p> <p><math>h \cos 30(k^2 - 1) = 4r \sin 30(k + 1)</math></p> <p><math>(k - 1) = \frac{4r}{h} \tan 30</math></p> <p><math>k = \frac{4r}{h\sqrt{3}} + 1 \quad *</math></p>	<p>M1A1, M1A1</p> <p>A1ft</p> <p>A1</p>

Question Number	Scheme	Marks
<b>Alt 2</b>	<p>Find <math>\bar{x}</math> first</p> $M(0) - \frac{1}{4}h + \frac{k^2h}{4} = (1+k)\bar{x}$ $\bar{x} = \frac{h(k-1)}{4} \quad \text{oe}$ <p>Then suspend: <math>\frac{\bar{x}}{r} = \tan 30</math></p> $\frac{h(k-1)}{4r} = \frac{1}{\sqrt{3}} \quad (\text{or } \tan 30)$ $k = \frac{4r}{h\sqrt{3}} + 1 \quad *$	<p>M1 A1</p> <p>A1</p> <p>M1</p> <p>A1ft</p> <p>A1</p>

**M1** Finding the distance of the c of m from  $O$  by using the angle given. Must use tan.

**A1** Obtaining  $\frac{r}{\sqrt{3}}$  (no approx allowed)

**M1** Forming a moments equation using the three known distances; mass ratio only needed – do not penalise use of incorrect formulae

**A1** LHS correct

**A1ft** RHS correct for their distance

**A1cao** Obtaining the GIVEN answer

#### **ALT 1 Taking moments about A**

**M1** Attempting the LHS – must have two appropriate terms inc the necessary resolution

**A1** Correct LHS

**M1** Attempting the RHS – must have two appropriate terms inc the necessary resolution

**A1** Correct RHS

**A1ft** Collecting the terms and cancelling  $Mg$

**A1cao** Completing to the GIVEN answer

#### **ALT 2 Find $\bar{x}$ first**

**M1** First M mark on e-PEN: Attempting an equation to find  $\bar{x}$  in terms of  $h$  and  $k$  - mass ratio as above

**A1** First A mark on e-PEN: Correct equation

**A1** Second A mark on e-PEN: Correct expression for  $\bar{x}$  (as shown or equivalent)

**M1** Second M mark on e-PEN: Using  $\frac{\bar{x}}{r} = \tan 30$  (LHS either way up)

**A1ft** Third A mark on e-PEN: Substitute their  $\bar{x}$ ; LHS must be the correct way up

**A1cao** Final A mark on e-PEN: Obtaining the GIVEN answer

Question Number	Scheme	Marks
<b>6 (a)</b>	$T_A = \frac{20x}{2.5} (=8x) \quad T_B = \frac{18(2-x)}{1.5} (=12(2-x))$ $\frac{20x}{2.5} = \frac{18(2-x)}{1.5}$ $x = \frac{12}{10} = 1.2$ $AO = 3.7 \text{ m}$	M1A1 A1 A1ft (4)
<b>(b)</b>	$\frac{18(0.8-y)}{1.5} - \frac{20(1.2+y)}{2.5} = 0.5\ddot{y}$ $-40y = \ddot{y} \therefore \text{SHM (or } \ddot{y} = (-20/m)y$	M1A1A1 A1cso (4)
<b>(c)</b>	(Max) speed = $\frac{6}{0.5} = 12 \text{ m s}^{-1}$ $\omega = \sqrt{40} = 2\sqrt{10}$ $12 = a \times 2\sqrt{10}$ $a = \frac{6}{\sqrt{10}} \text{ or } \frac{3\sqrt{10}}{5} \text{ m (accept 1.897... ie 1.9, 1.90 or better)}$	B1 B1ft M1 A1ft (4)
<b>(d)</b>	$1.2 = a \sin \omega t$ $t = \frac{1}{2\sqrt{10}} \sin^{-1} \left( \frac{1.2\sqrt{10}}{6} \right)$ $t = 0.1082... \text{ s (Accept 0.11 or better)}$	M1(their $a, \omega$ ) M1(must use radians) A1cso (3) [15]

- (a) M1** Using Hooke's law to find **both** tensions and equating them. The extension in *BP* can be used instead of the extension in *AP*. ALT: Use both extensions and use  $e_a + e_b = 2$  later
- A1** Correct equation
- A1** Correct value found for either extension
- A1ft** Correct length for *AO*; follow through their extension
- (b) M1** Forming an equation of motion at a general point. Difference of 2 tensions, both including the variable. Use of  $a$  instead of  $\ddot{x}$  can score M1A1A0A0 max (ie an A error)
- A1 A1** A1A1 fully correct; A1A0 one error May have  $m$  instead of 0.5 Extensions measured from *O*
- A1cso** A correct simplified equation. Any equivalent form, including having  $m$  instead of 0.5. There must be a concluding statement.
- (c) B1** Correct speed following impulse Can be awarded if seen in (b) or (d)
- B1ft** Correct value of  $\omega$ ; must be numerical. FT from (b) Can be awarded if seen in (b) or (d)
- M1** Using  $v_{\max} = a\omega$  (their values). By energy – equation must have all terms
- A1ft** Correct value of  $a$  any equivalent form including decimals. Follow through their  $\omega$
- (d) M1** Using  $y = a \sin \omega t$  with their  $a$  and  $\omega$  If  $y = a \cos \omega t$  is used there must be some indication of moving from the time obtained to the required time.
- M1** Solving their equation to find a time. **Must** use radians
- A1cso** Correct time, min 2 sf.  $\omega$  and  $a$  must have been obtained from correct work.



Question Number	Scheme	Marks
<b>7 (a)</b>	$\frac{1}{2}mv^2 - \frac{1}{2}m\frac{rg}{4} = mgr(1 - \cos\theta)$ $v^2 = \frac{rg}{4}(9 - 8\cos\theta) \quad *$	M1A1A1 A1 (4)
<b>(b)</b>	$(R) + mg\cos\theta = \frac{mv^2}{r}$ $R = 0 \quad mg\cos\alpha = \frac{mg}{4}(9 - 8\cos\alpha)$ $12\cos\alpha = 9$ $\cos\alpha = \frac{3}{4} \text{ or } 0.75$	M1A1 DM1 A1 (4)
<b>(c)</b>	$\text{Initial vert comp of speed} = \sqrt{\frac{3g}{8}}\sin\alpha = \sqrt{\frac{3g}{8}} \times \frac{\sqrt{7}}{4} (=1.2679\dots)$ $\frac{7}{8} = 1.2679\dots t + \frac{1}{2}gt^2$ $7 = 10.143\dots t + 39.2t^2$ $39.2t^2 + 10.143\dots t - 7 = 0$ $t = \frac{-10.143 \pm \sqrt{10.143^2 + 4 \times 7 \times 39.2}}{2 \times 39.2}$ $t = 0.3125\dots$ $\text{Horiz speed} = \sqrt{\frac{3g}{8}}\cos\alpha = \frac{1}{4}\sqrt{\frac{27g}{8}}$ $AC = \frac{1}{4}\sqrt{\frac{27g}{8}} \times 0.3125 + r\sin\alpha = 0.4493 + 0.3307 = 0.78 \text{ m}$	M1A1 M1 DM1 A1 M1A1cso (7) [15]

**(a) M1** Attempting an energy equation. 2KE terms needed and a PE term.

Award if mass missing throughout, but **not** for use of  $v^2 = u^2 + 2as$

**A1** KE terms correct (and subtracted) Mass not needed if M mark earned

**A1** PE correct Again, mass not needed if M mark earned

**A1cso** Obtaining the GIVEN answer

**(b) M1** Attempting an equation of motion along the radius. Accel in either form,  $(\pm)R$  may be included.

**A1** Correct equation, with or without  $(\pm)R$

**DM1** Set  $R = 0$  and substitute for  $v$

**A1**  $\cos\alpha = 3/4$  obtained

- (c) **M1** Attempting the initial vertical component of the speed  
**A1** Correct vertical component - decimal or exact  
**M1** Using  $s = ut + \frac{1}{2}at^2$  to form a quadratic in  $t$ , with *their* vertical speed and attempt at the vertical distance **Must** satisfy  $0.5 < \text{distance} < 1$   
**DM1** Solving their quadratic; formula must be shown (and correct) if answer is incorrect, but allow with  $+\sqrt{\dots}$  instead of  $\pm\sqrt{\dots}$   
**A1** Correct  $t$ . Give by implication if stored on a calculator and final answer correct  
 Second solution need not be shown; ignore any shown  
**M1** Using the horizontal speed and completing to obtain the required distance.  
**A1**  $AC = 0.78$  **must** be 2 sf.

**ALT for (c):**

- M1A1** As main method above  
**M1** Use the horizontal speed and distance travelled as a projectile to get an expression for  $t$  and substitute in  $s = ut + \frac{1}{2}at^2$  Vertical distance must be between 0.5 and 1  
**DM1** Solve their quadratic - see above  
**A1** Correct (projectile) distance  
**M1A1** As main method above

**7(c) Using energy etc:**

<b>M1</b>	Using energy to get the speed at the floor. Can be from the top or the point of leaving the surface
<b>A1</b>	Correct speed at floor
<b>M1</b>	Using the horizontal component of the speed and Pythagoras to obtain the vertical component at the floor
<b>M1</b>	Using $v = u + at$ vertically to get $t$
<b>A1</b>	Correct $t$
<b>M1A1</b>	Complete as main method

**Other alternative Methods****Question 4(a) by definite integration**

$900 \frac{dv}{dt} = \frac{63000}{kt^2}$	M1
$\int_0^{10.5} dv = \int_1^4 \frac{70}{kt^2} dt$	
$[v]_0^{10.5} = \left[ -\frac{70}{kt} \right]_1^4$	DM1A1 Integration, limits not needed
$10.5(-0) = -\frac{70}{4k} + \frac{70}{k}$	M1 Substitute limits
$k = 5$	A1 Correct value
$\int_0^v dv = \int_1^t \frac{14}{t^2} dt$	A1 Integrate again with limits as shown
$v = 14 - \frac{14}{t} \quad *$	A1 Obtain GIVEN answer

**OR:**

$900 \frac{dv}{dt} = \frac{63000}{kt^2}$	M1
$\int_0^v dv = \int_1^t \frac{70}{kt^2} dt$	
$[v]_0^v = \left[ -\frac{70}{kt} \right]_1^t$	DM1A1 Integration, limits not needed
$v = \frac{70}{k} \left[ -\frac{1}{t} \right]_1^t = \frac{70}{k} \left( 1 - \frac{1}{t} \right)$	M1 Substitute limits and $v = 10.5, t = 4$
$k = 5$	A1 Correct value
$v = \frac{70}{5} \left( 1 - \frac{1}{t} \right)$	A1 substitute
$v = 14 - \frac{14}{t} \quad *$	A1 Obtain GIVEN answer

**Question 6(c) by reference circle****M1** Finding the required angle in radians.**M1** Using the period  $\left( \frac{2\pi}{\omega} \right)$  and their angle to find the required time.**A1** Correct time.

